Dissipation & adiabatic quantum computation

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Outlook

- **Adiabatic quantum computation**
  theoretical framework
  superconducting qubits

- **A prototypical scenario**
  the quantum Ising chain
  coupling with an environment

⇓

Quantum annealing with dissipation

Adiabatic Quantum Computation

A class of procedures for solving *optimization problems* with quantum computers.

Basic strategy:

- **design** a *problem Hamiltonian* H_P whose ground state encodes the solution of an optimization problem

- **prepare** the known ground state of a simple Hamiltonian H_0

- **interpolate** slowly

$$H(s) = \left[1 - s\right] H_0 + s\, H_P$$

$$s(t) \in [0, 1]$$

$$s(0) = 0;\ \ s(T) = 1$$

E. Fahri, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, D. Preda, *Science* ***292**, 472 (2001)*
G. E. Santoro, R. Martonak, E. Tosatti, R. Car, *Science* ***295**, 2427 (2002)*

Adiabatic Quantum Computation

$$H(s) = \big[1 - s\big] H_0 + s\, H_P$$

$$s(t) \in [0,1]$$

$$s(0) = 0;\ s(T) = 1$$

The interpolation has to be done *slowly*.

According to the **adiabatic theorem**, the time *T* has to be:

$$T \gg \Gamma^2/\Delta_{\min}^2 \qquad \text{with} \;\; \Gamma^2 = \max_{s\in[0,1]} \big\| \big[\dot{H}(s)\big]^2 \big\|$$

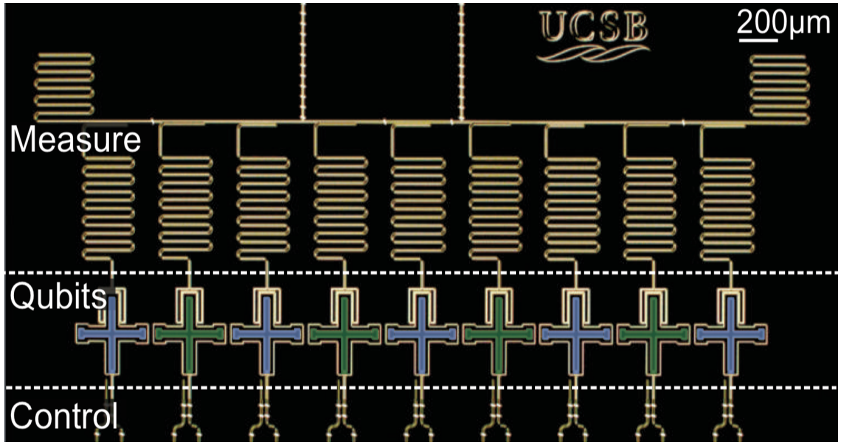
How big is $\Delta_{\min}$?

$\Delta_{\min} \gtrsim 1/\mathrm{poly}(N)$ ⟶ efficient quantum algorithm

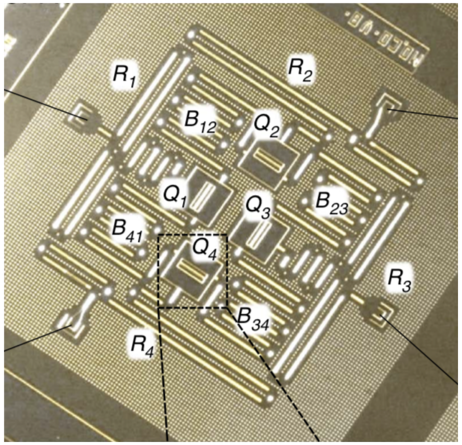
$\Delta_{\min} \sim 1/\exp(N)$ ⟶ inefficient quantum algorithm

Hard problems (NPC) are equivalent to finding the gs of *Ising-like spin-glass* Hamiltonians. F. Barahona, *J. Phys. A* ***15****, 3241 (1982)*

AQC in real quantum devices?



Martinis lab (UCSB / Google)



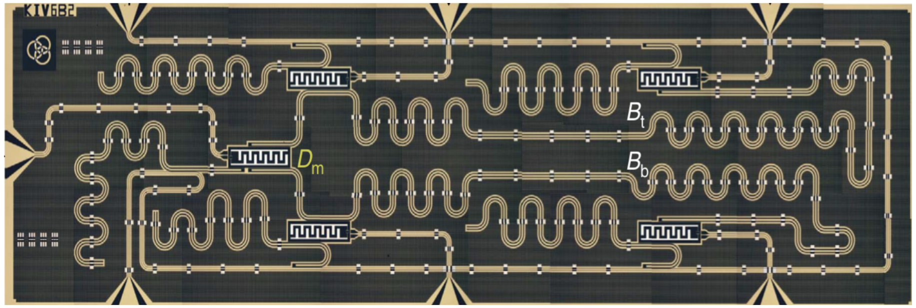
IBM lab (NY)

Superconducting circuits

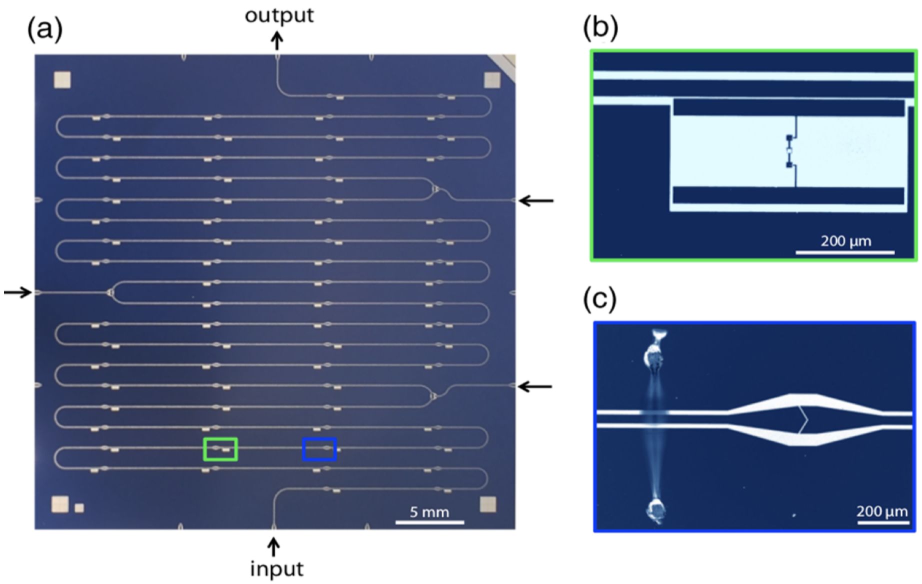
→ quantum simulation

→ quantum annealing

→ quantum computing

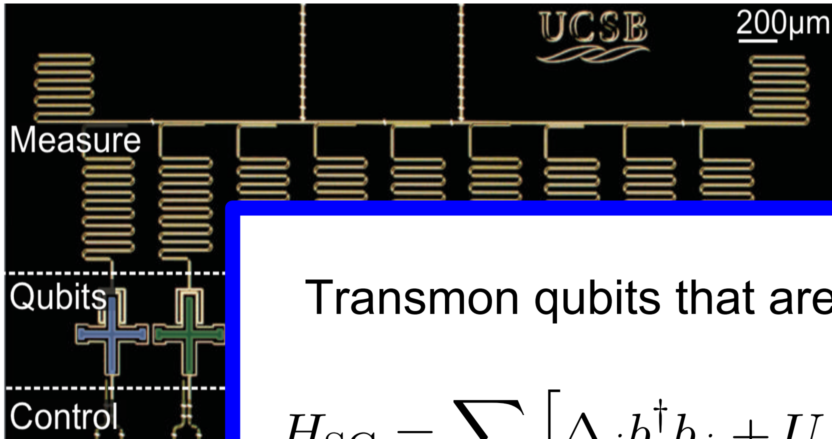


Di Carlo lab (TU Delft)



Houck lab (Princeton)

AQC in real quantum devices?



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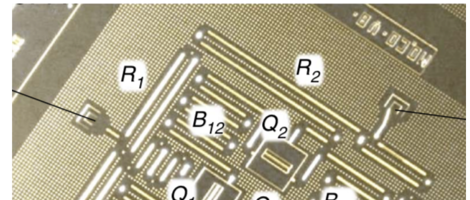


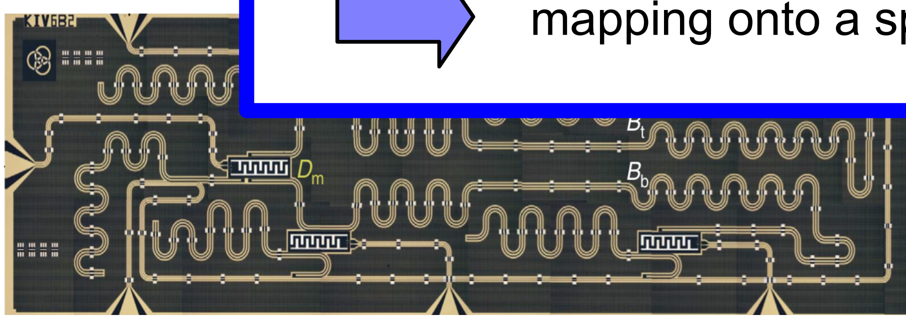
Superconducting circuits

simulation
annealing
computing

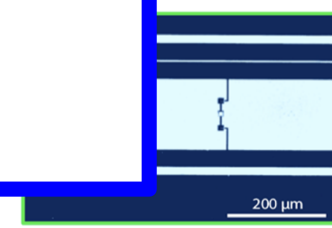
Transmon qubits that are capacitively coupled:

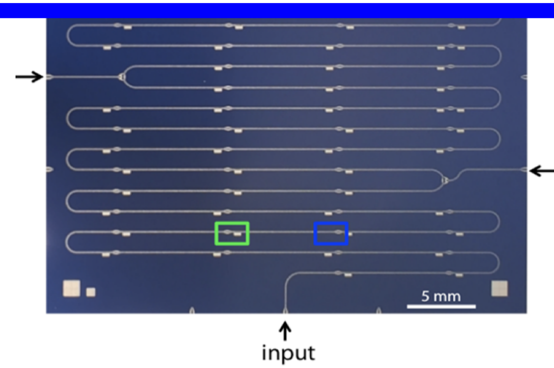
$$H_{\mathrm{SC}} = \sum_j \left[\Delta_j b_j^\dagger b_j + U_j b_j^\dagger b_j^\dagger b_j b_j + \left(\Omega_j b_j^\dagger + J_j b_j^\dagger b_{j+1} + \mathrm{H.c.} \right) \right]$$

mapping onto a spin-chain Hamiltonian

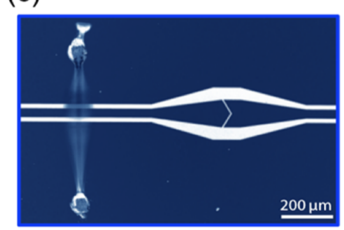


Di Carlo lab (TU Delft)





(c)



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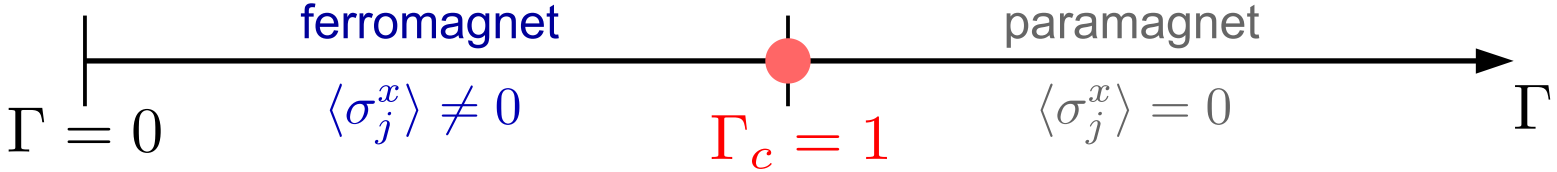
Quantum annealing with dissipation

(Quasi-)adiabatic dynamics in the Ising chain

Consider a *quantum Ising chain* with a slowly varying field:

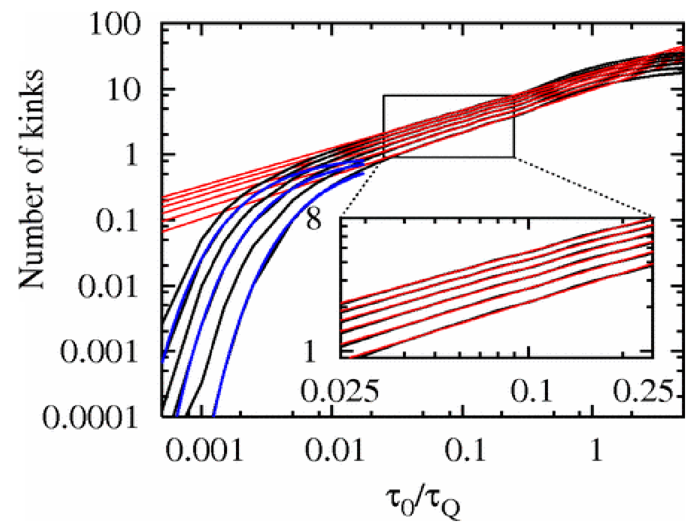
$$H(t) = -\sum_j \sigma_j^x \sigma_{j+1}^x - \Gamma(t) \sum_j \sigma_j^z \qquad \Gamma(t) = -t/\tau, \quad t \in (-\infty, 0]$$



When crossing the phase transition, the gap closes: $\Delta \sim |\Gamma - \Gamma_c|^{z\nu}$

Is it possible to estimate the scaling of the generated defects with τ ?

A. Polkovnikov, *PRB **72**, 161201(R) (2005)*
W. Zurek, U. Dorner, P. Zoller, *PRL **95**, 105701 (2005)*
J. Dziarmaga, *PRL **95**, 245701 (2005)*



Adiabatic dynamics & noise

While, in the unitary case, the number of defects decreases on increasing the annealing time, the ***noise*** should dominate for long annealing times.

- Thermal fluctuations

  M. H. Amin, P. J. Love, C. J. Truncik, *PRL **100**, 060503 (2008)*
  D. Patanè, A. Silva, L. Amico, R. Fazio, G. E. Santoro, *PRL **101**, 175701 (2008)*

- Spatially correlated noise

  P. Nalbach, S. Vishveshwara, A. A. Clerk, *PRB **92**, 014306 (2015)*

- Enhanced performances by thermally assisted tunneling

  S. Boixo et al., *Nat. Commun. **7**, 10327 (2016)*
  K. Kechedzhi, V. N. Smelyanskiy, *PRX **6**, 021028 (2016)*
  V. N. Smelyanskiy et al., *PRL **118**, 066802 (2017)*
  Jiang, Smelyanskiy, Isakov, Boixo, Mazzola, Troyer, Neven, *PRA **95**, 012322 (2017)*

- Landau-Zener model coupled to an environment

  P. Ao, J. Rammer, *PRL **62**, 3004 (1989)*
  M. Wubs, K. Saito, S. Kohler, P Hänggi, Y. Kayanuma, *PRL **97**, 200404 (2006)*
  D. Zueco, P. Hänggi, S. Kohler, *NJP **10**, 115012 (2008)*
  S. Javanbakht, P. Nalbach, M. Thorwart, *PRA **91**, 052103 (2015)*
  L. Arceci, S. Barbarino, R. Fazio, G. E. Santoro, *PRB **96**, 054301 (2017)*

Theoretical framework

A state of an open quantum system is described by a density matrix $|\rho(t)\rangle\rangle$ in a Hilbert space $\mathcal{H}$. Its evolution is generated by a ***master equation***, which may take the form:

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \sum_j \kappa_j \left[L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right]$$

coherent dynamics
(hopping, interaction & driving, …)

Coupling to external bath
(incoherent pumping, losses, …)

Lindblad master equation

L_j: system "jump" operators
(or Lindblad operators)

Approximations:

- *Born* (weak system-bath coupling)
- *Markov* (neglect memory effects)
- *Secular* (neglect fast oscillations)

H.-P. Breuer, F. Petruccione, "*The theory of open quantum systems*" (*Oxford Univ. Press 2002*)

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↓

Quantum annealing with dissipation

M. Keck, S. Montangero, G. Santoro, R. Fazio, DR, New J. Phys. ***19****, 113029 (2017)*

A simple (exactly solvable) example

The benchmark Hamiltonian is again an **Ising chain**:

$$H(t) = -\sum_j \sigma_j^x \sigma_{j+1}^x - \Gamma(t) \sum_j \sigma_j^z \qquad \Gamma(t) = -t/\tau, \quad t \in (-\infty, 0]$$

mappable into a free-fermionic model through the *Jordan-Wigner transform*

$$\Rightarrow \quad H(t) = -\sum_j \left\{ \left(c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1}^\dagger + \text{h.c.} \right) + 2\Gamma(t) c_j^\dagger c_j \right]$$

can be easily diagonalized with a *Fourier transform*

followed by a *Bogoliobuv transform*

$$\Rightarrow \quad H = \sum_{k>0} H_k \quad \begin{matrix} \nearrow \{ |0\rangle, |1_k, 1_{-k}\rangle \} \\ \searrow \{ |1_k\rangle, |1_{-k}\rangle \} \end{matrix}$$

E. Lieb, T. Schultz, D. Mattis, *Ann. Phys.* ***16**, 407 (1961)*
P. Pfeuty, *Ann. Phys.* ***57**, 79 (1970)*

A simple (exactly solvable) example

Dissipation is modeled through memoryless local jump operators:

$$L_j^{(1)} = c_j^\dagger$$ incoherent pumping

$$L_j^{(2)} = c_j$$ incoherent losses

$$L_j^{(3)} = c_j^\dagger c_j$$ dephasing bath

T. Prosen, *NJP* ***10****, 043026 (2008)*

V. Eisler, *J. Stat. Mech. (2011) P06007*

B. Horstmann, J.I. Cirac, G. Giedke, *PRA* ***87****, 012108 (2013)*

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→ For incoherent pumping/decay, the Liouvillian is still quadratic.

$$\partial_t \rho = -i[H, \rho] + \kappa \sum_j \left[L_j \rho L_j^\dagger - \tfrac{1}{2}\{L_j^\dagger L_j, \rho\} \right]$$

Dissipative terms do not mix the various momentum modes but violate the fermionic parity: $\rho(t) = \bigotimes_k \rho_k(t)$

$\{|0\rangle, |1_k\rangle, |1_{-k}\rangle, |1_k, 1_{-k}\rangle\}$

T. Prosen, *NJP* ***10**, 043026 (2008)*
V. Eisler, *J. Stat. Mech. (2011) P06007*
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→ For a dephasing bath, one can focus on certain observables (two-point correlation functions) and write a closed set of differential equations.

$$\frac{d}{dt}\begin{bmatrix} \vec{F} \\ \vec{G} \\ \vec{I} \\ \vec{K} \end{bmatrix} = \mathcal{M}_{4N\times 4N}\begin{bmatrix} \vec{F} \\ \vec{G} \\ \vec{I} \\ \vec{K} \end{bmatrix}$$

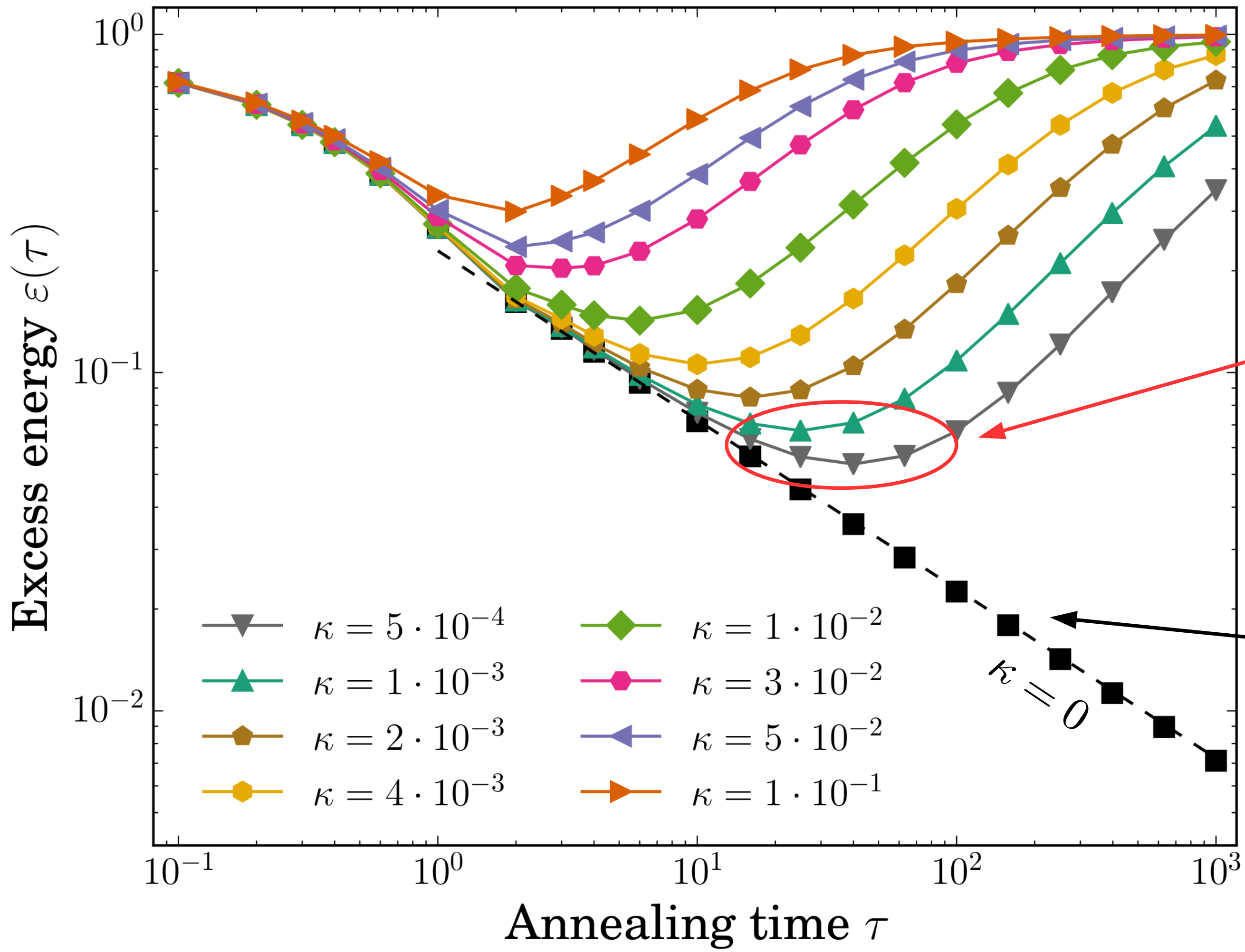
$$[\vec{F}]_r = \langle c_m^\dagger c_n \rangle\,, \quad [\vec{G}]_r = \langle c_m c_n^\dagger \rangle$$

$$[\vec{I}]_r = \langle c_m^\dagger c_n^\dagger \rangle\,, \quad [\vec{K}]_r = \langle c_m c_n \rangle$$

T. Prosen, *NJP* ***10****, 043026 (2008)*
V. Eisler, *J. Stat. Mech. (2011) P06007*
B. Horstmann, J.I. Cirac, G. Giedke, *PRA* ***87****, 012108 (2013)*

Quantum annealing with incoherent pumping

A) Fermionic quadratic model + incoherent pumping $L_j^{(1)} = c_j^\dagger$



optimal working point

KZ scaling:

$$\varepsilon(t) \sim 1/\tau^\gamma$$

$(\gamma = 1/2)$

$$\varepsilon(t) = \frac{1}{L}\left\{ \mathrm{Tr}\left[H(t)\,\rho(t)\right] - \langle \psi_0(t) | H(t) | \psi_0(t) \rangle \right\}$$

Optimal working point

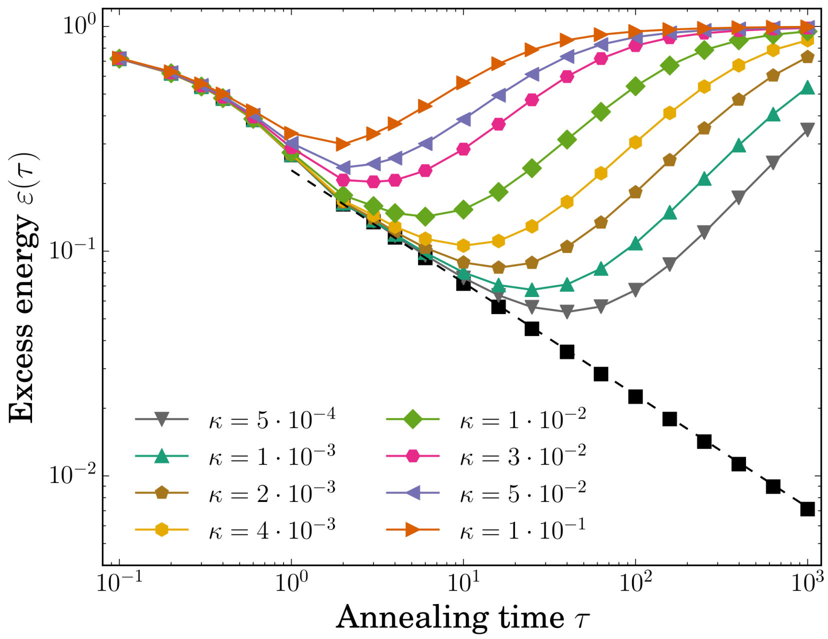
Non monotonic behavior: a competing effect between KZ & dissipation …

→ *optimal working point*

$$\varepsilon_{\rm opt} \sim \kappa^{1/3}$$

$$\tau_{\rm opt} \sim \kappa^{-2/3}$$

Can be understood by a supposing a substantial independence of the role played by dissipation, with respect to the KZ mechanism.



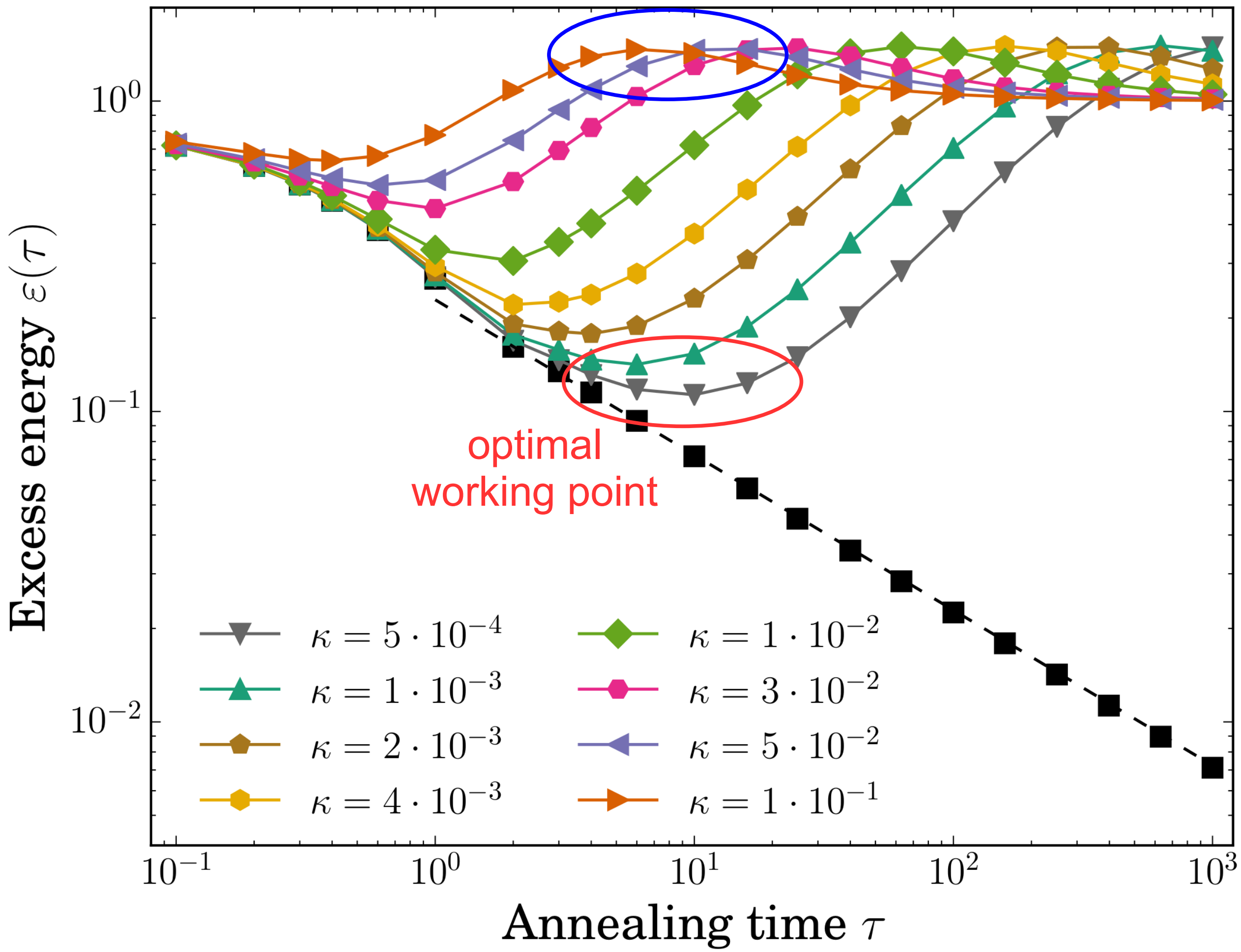
The incoherent coupling to an external bath acts uniformly and irrespective of the adiabaticity condition ruled by the ground-state energy gap:

$$\mathcal{N}_{\rm tot} = \mathcal{N}_{\rm KZ} + \mathcal{N}_{\rm inc} = \frac{1}{2\pi\sqrt{2}}\tau^{-1/2} + \frac{1}{2}\kappa\tau \qquad \partial_\tau \mathcal{N}(\tau)|_{\tau_{\rm opt}} = 0$$

Similar observations in: A. Dutta, A. Rahmani, A. del Campo, *PRL* ***117****, 080402 (2016)*

Quantum annealing with incoherent decay

B) Fermionic quadratic model + incoherent decay $L_j^{(2)} = c_j$

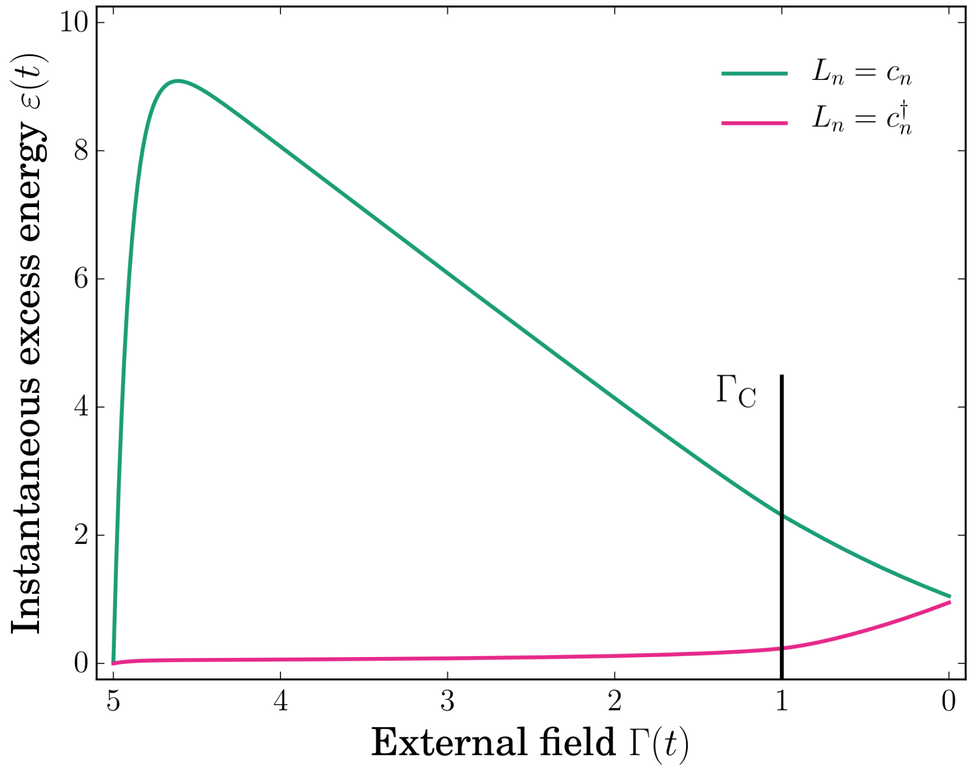


overshooting point:

defects become larger than those reached for infinitely slow annealing.

Intrinsically related to the coupling with a bath, which drives the system toward the steady state according to the Liouvillian dynamics.

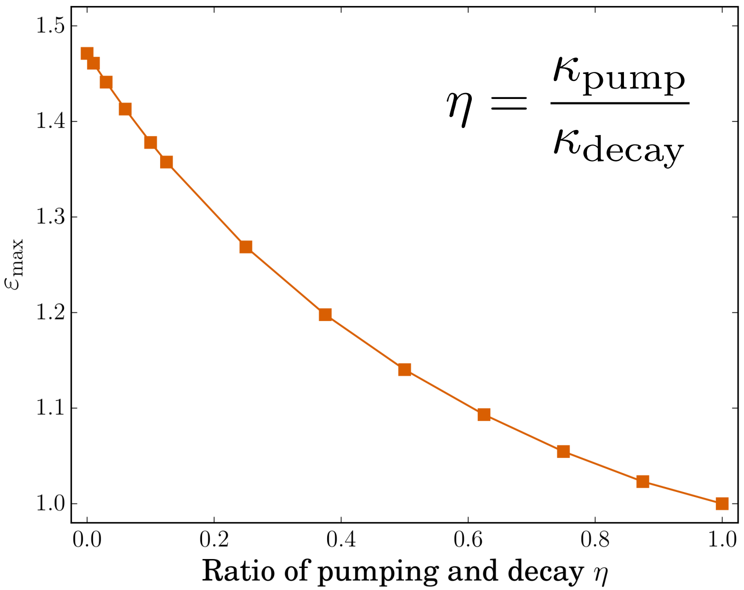
Overshooting point



$$|\psi_0(t \to -\infty)\rangle = |11\cdots 1\rangle$$

→ with *pumping* $\varepsilon(t)$ mostly increases in the final part of the protocol.

→ with *decay*, the steady state is initially very far from from $|\psi_0\rangle$).

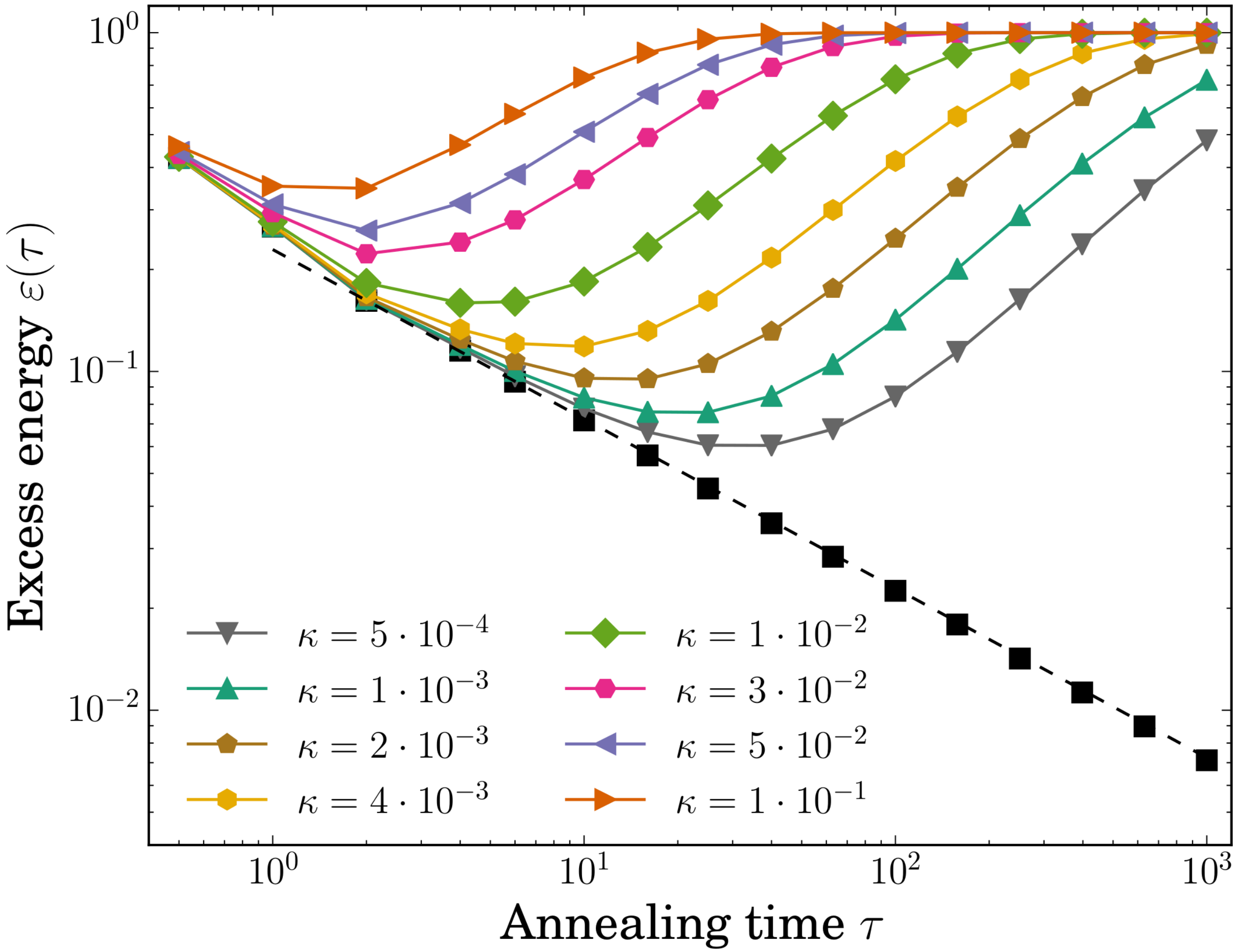


For η < 1 the instantaneous steady-state energy decreases from a positive value to 0 (→overshooting).

For η = 1 it is constant equal to 0, while for η > 1 it approaches 0 from below (→ prevents overshooting).

Quantum annealing with dephasing

C) Fermionic quadratic model + dephasing $L_j^{(3)} = c_j^\dagger c_j$



→ qualitatively similar to incoherent pumping (with a little worsening of AQC performance)

→ same scaling of the optimal working point

→ no overshooting

Conclusions

- ✔ AQC with ***Markovian & local dissipation***
- ✔ **Optimal working point** & overshooting
  KZ effects & dissipation act **incoherently**

*M. Keck et al., NJP **19**, 113029 (2017)*