Thermal drag in capacitively-coupled metallic islands

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Coulomb drag

- Two electrically isolated conductors

- Only upper conductor is biased
- Current is **dragged** in lower if they are coupled via the **Coulomb interaction** (energy and momentum transfer)

- Layered systems (2DEG, graphene)
- 1D wires
- QH edge states
- ...
Coulomb drag

0D, single-electron systems

- Single level QD, broken detail balance (energy-dependent lead couplings)
  Sánchez, Lopez, Sánchez, Büttiker (2010)
  Moldoveanu, Tanatar (2009)

Experiment, QD

- Graphene-based QD
  Volk et al. (2015)
  Bischoff et al. (2016)

- Lithographically-patterned QD
  Keller et al. (2016)

- Cotunnelling is crucial
  Kaasbjerg, Jauho (2016)
Coulomb drag

- Energy harvesting from thermal/voltage fluctuations, thermocouple heat engine

\[ T_1 = T_2 \]

- energy-dependence and asymmetry of lead coupling essential

Sánchez, Büttiker (2011)
Sothmann, Sánchez, Jordan, Büttiker (2012)
Sánchez, Sothmann, Jordan, Büttiker (2013)
Sothmann, Sánchez, Jordan (2015)
Daré, Lombardo (2017)
Whitney, Sánchez, Haupt, Splettstoesser (2016)

Hartmann, Pfeffer, Höfling, Kamp, Wordchech (2015)
Thierschmann, Arnold, Mittermüller, Maier, Heyn, Hansen, Buhmann, Molemkamp (2015)
Thermal drag

* Capacitively-coupled **metallic islands**, lead coupling energy-independent

Koski, Kutvonen, Khaymovich, Ala-Nissila, Pekola (2015)
Capacitively-coupled metallic islands

- **Coulomb-blockade regime**
- **Lead-island couplings are energy-independent**
- **Sequential tunneling**

- No dragged charge current, even for \( R_{L2} \neq R_{R2} \)
- Finite dragged heat current for \( R_{L2} \neq R_{R2} \)
Capacitively-coupled metallic islands

**Electrostatic energy**

\[
U(n_1, n_2) = E_{C,1} (n_1 - n_{x1})^2 + E_{C,2} (n_2 - n_{x2})^2 + E_I (n_1 - n_{x1}) (n_2 - n_{x2})
\]

\[
n_{x1} = V_{g1} \frac{C_g}{e}
\]

\[
n_{x2} = V_{g2} \frac{C_g}{e}
\]

Inter-island interaction energy controlled by \( C_I \)

**Electrostatic energy change for transitions in lower island**

\[
\delta U_2(n_1, n_2) = U(n_1, n_2 + 1) - U(n_1, n_2)
\]

Depends on the charge state in island 1
Capacitively-coupled metallic islands

- Dragged heat current results from **energy transferred** from drive circuit, through this mechanism.

\[
\begin{align*}
\delta U_2(0,0) &= E_C(1 - 2n_{x2}) - E_1 n_{x1}, \\
\delta U_2(1,0) &= E_C(1 - 2n_{x2}) + E_1 (1 - n_{x1}),
\end{align*}
\]

- Heat current associated to this processes: \( E_1 \)

- Heat currents are modulated through gate voltages
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**Dependence on gate voltage**

- Width independent of \( E_I \), but controlled by temperature
- Maximum occurs in the symmetric energy configuration

\[
E_I = 0.4E_C \\
k_B T = 0.05E_C \\
k_B \Delta T = 0.08E_C \\
R_{L1} = R_{R1} = R_{L2} = 5R_Q \\
R_{R2} = 10R_Q
\]

\[
I_0^{(h)} = e^2/(4C^2R)
\]
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Sequential tunnelling, small bias

expansion in $\Delta T/T$

$$I_{\text{drag}}^{(h)} = \frac{\xi R_\parallel}{6 e^2 R} \left[ \frac{1}{R_{L2}} - \frac{1}{R_{R2}} \right] \text{csch} \xi \left[ 2 \xi \left( \frac{\pi^2}{4} + \xi^2 \right) \text{csch} \xi - \left( \frac{\pi^2}{2} + 3 \xi^2 \right) \text{sech} \xi \right] \left( k_B \Delta T \right)^2$$

$\checkmark$ Second order in $\Delta T$

$n_{x1} = n_{x2} = \frac{1}{2}$

$\xi = \frac{E_I}{4k_B T}$

expansion in $eV/E_C$

$$I_{\text{drag}}^{(h)} = \frac{\xi R_\parallel}{16 R} \left[ \frac{1}{R_{L2}} - \frac{1}{R_{R2}} \right] \text{csch} \xi \left[ \xi \text{csch} \xi - \text{sech} \xi \right] V^2$$

$\checkmark$ Second order in $V$
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- Sequential tunnelling, dependence on inter-island coupling

![Graph showing sequential tunnelling with curves for different values of \( k_B \Delta T \) and \( eV \).]

- Maximal for intermediate values
- Position of the maximum obtained from analytics
- Deviation due to \( T \) not small

\[ I_0^{(h)} = \frac{e^2}{4C^2R} \]

\[ E_I^{\text{max}} \approx 8.5k_B T \]

\[ E_I^{\text{max}} \approx 5.5k_B T \]
Capacitively-coupled metallic islands

- Sequential tunnelling, drag-drive comparison

- Temperature-biased case: \( I_{R2}^{(h)} < I_{R1}^{(h)} \)

- Voltage-biased case: \( I_{R2}^{(h)} > I_{R1}^{(h)} \)

- Larger heat current in the drag circuit for large interaction
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- Cotunnelling contributions, large bias

- Non-trivial contributions
- Quadratic dependence on $V$ and $\Delta T$
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- Energy-dependent couplings: superconductor

\[ \Delta \ll E_C \]

\[ n_{x2} < \frac{1}{2} \]

- Heat dragged to the right

\[ n_{x2} \]
Conclusions

- Thermal drag in capacitively-coupled metallic islands

  - Sequential tunneling regime
  - Co-tunneling contributions

  - Analytic expressions for heat currents for small biases

  - Dependence on the inter-islands coupling

  - Energy-Dependent island-lead couplings