# Thermal drag in capacitively-coupled metallic islands


Fabio Taddei

NEST, NANO-CNR & Scuola Normale Superiore, Pisa, Italy

Bibek Bhandari (SNS & NEST, Pisa, Italy)
Paolo Erdman (SNS & NEST, Pisa, Italy)
Giuliano Chiriacò (Columbia University, New York, USA)
Rosario Fazio (ICTP Trieste & SNS Pisa, Italy)



January 29th 2018, Les Houches
Frank Hekking memorial workshop


CNRNANO PISA NEST

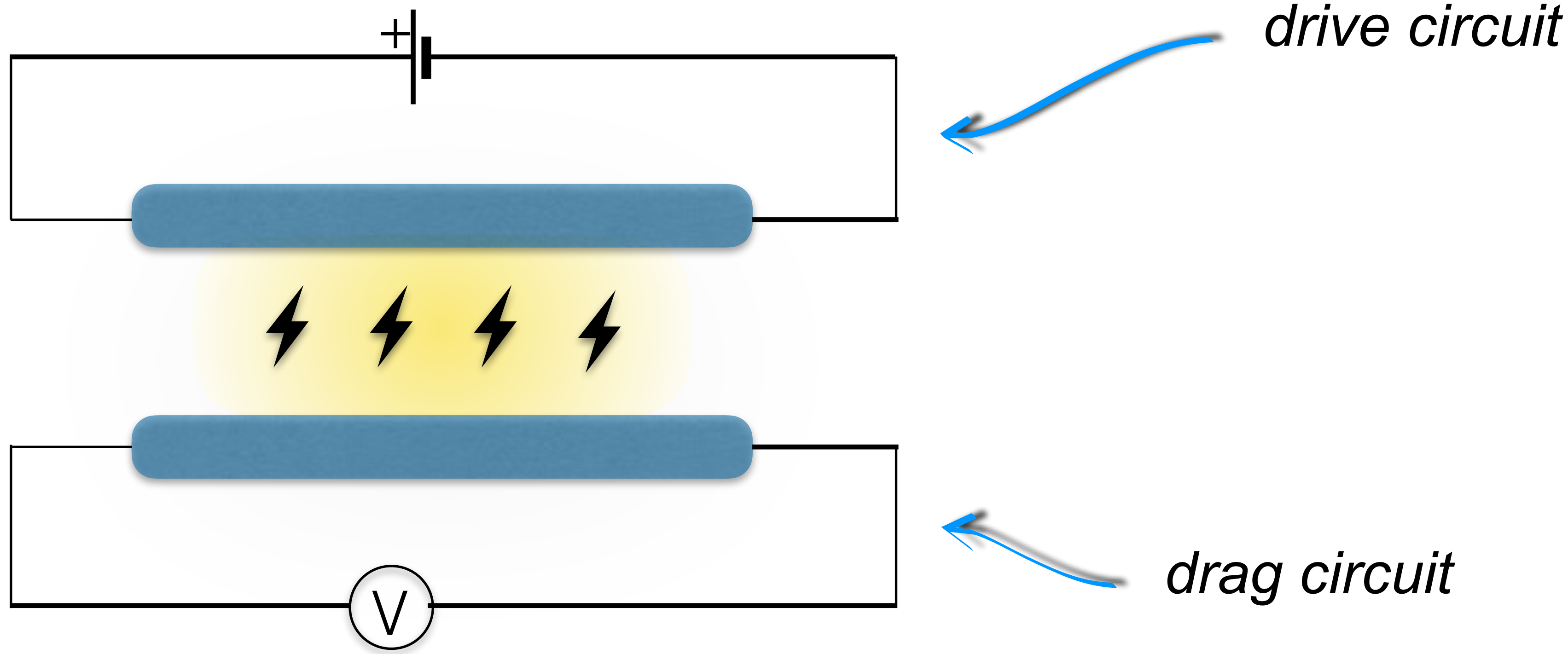
NEST SCUOLA NORMALE SUPERIORE PISA

National Enterprise for nanoScience and nanoTechnology

NEST

# Coulomb drag

➢ Two electrically isolated conductors




Pogrebinskii (1977)

✱ only upper conductor is biased
✱ current is **dragged** in lower if they are coupled via the **Coulomb interaction** (energy and momentum transfer)

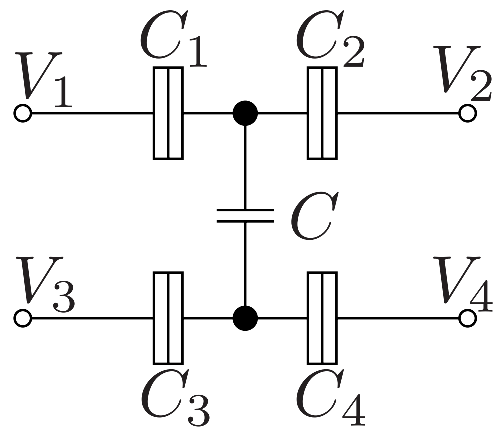
→ layered systems (2DEG, graphene)
→ 1D wires
→ QH edge states
→ …

Review: Narozhny, Levchenko (2016)

National Enterprise for nanoScience and nanoTechnology
NEST

# Coulomb drag

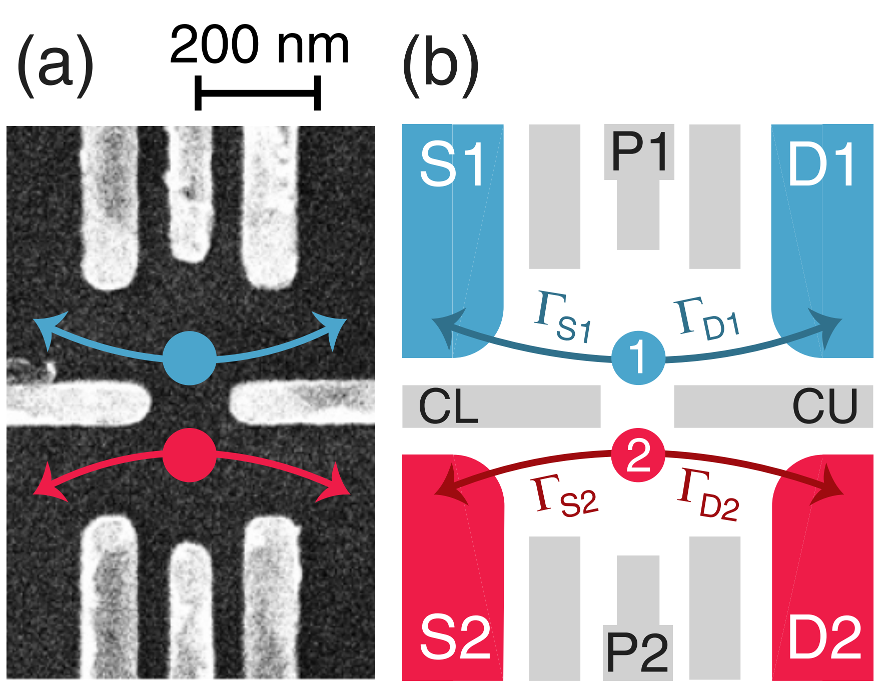
## ➢ 0D, single-electron systems




- sigle level QD, broken detail balance (energy-dependent lead couplings)

Sánchez, Lopez, Sánchez, Büttiker (2010)
Moldoveanu, Tanatar (2009)

## ➢ Experiment, QD




Keller et al. (2016)

- graphene-based QD

Volk et al. (2015)
Bischoff et al. (2016)

- lithographically-patterned QD

Keller et al. (2016)

- cotunnelling is crucial

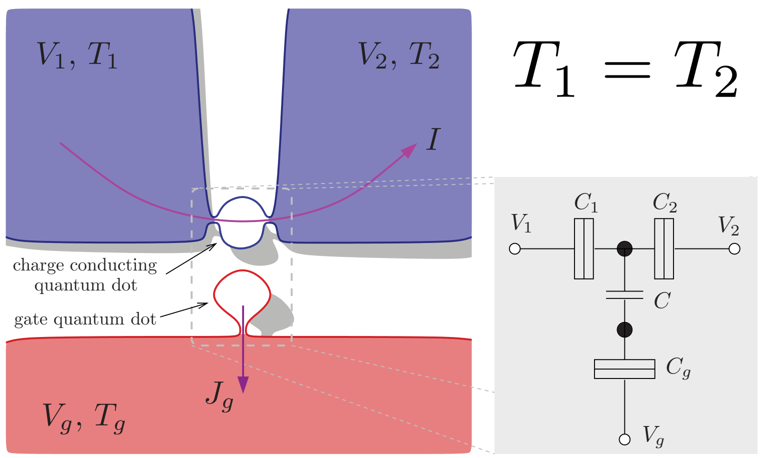
Kaasbjerg, Jauho (2016)

# Coulomb drag

➢ Energy harvesting from thermal/voltage fluctuations, thermocouple heat engine

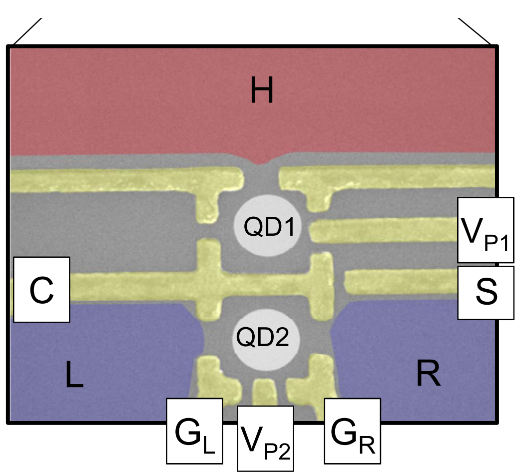
Sánchez, Büttiker (2011)
Sothmann, Sánchez, Jordan, Büttiker (2012)
Sánchez, Sothmann, Jordan, Büttiker (2013)
Sothmann, Sánchez, Jordan (2015)
Daré, Lombardo (2017)

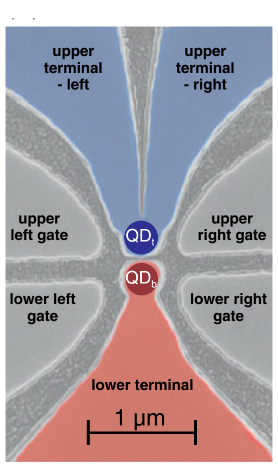
Whitney, Sánchez, Haupt, Splettstoesser (2016)




$$T_1 = T_2$$

✱ energy-dependence and asymmetry of lead coupling essential



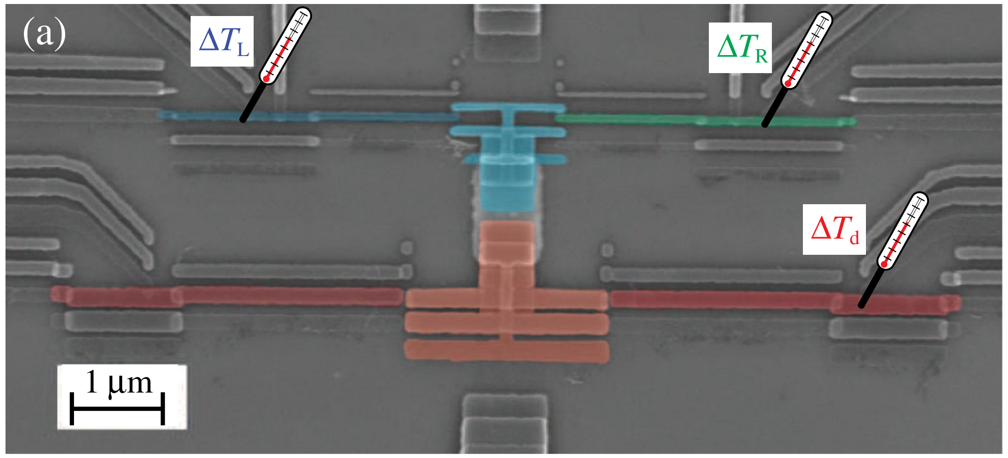

Hartmann, Pfeffer, Höfling, Kamp, Wordchech (2015)
Thierschmann, Arnold, Mittermüller, Maier, Heyn, Hansen, Buhmann, Molemkamp (2015)

# Thermal drag

dragged heat current

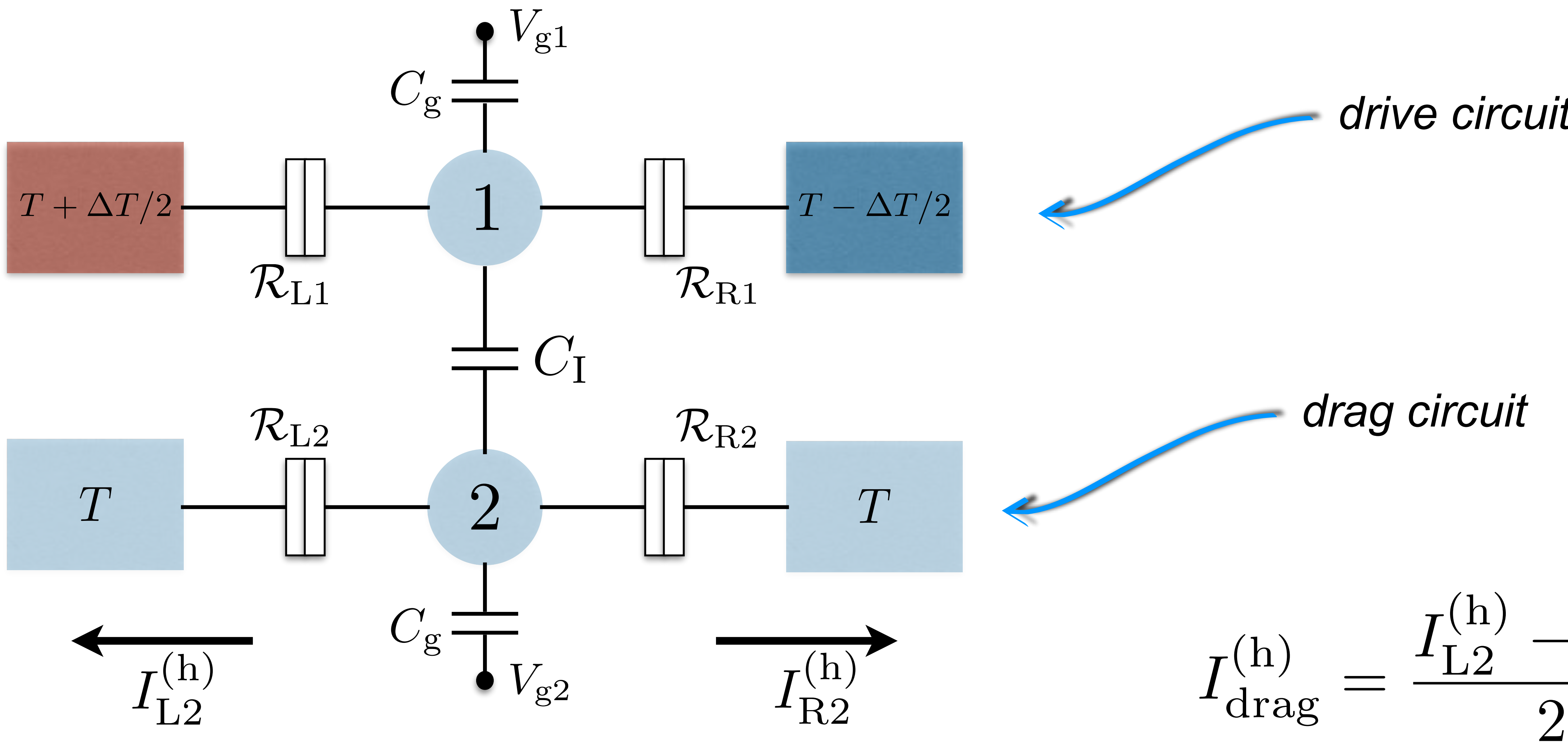
temperature bias

✱ Capacitively-coupled **metallic islands**, lead coupling energy-independent




Koski, Kutvonen, Khaymovich, Ala-Nissila, Pekola (2015)

# Capacitively-coupled metallic islands




$$I^{(h)}_{drag} = \frac{I^{(h)}_{L2} - I^{(h)}_{R2}}{2}$$

- Coulomb-blockade regime
- Lead-island couplings are **energy-independent**
- Sequential tunneling

☑ **No dragged charge** current, even for $\mathcal{R}_{L2} \neq \mathcal{R}_{R2}$

☑ **Finite dragged heat** current for

# Capacitively-coupled metallic islands

* Electrostatic energy

$$U(n_1,n_2) = E_{C,1}\left(n_1 - n_{x_1}\right)^2 + E_{C,2}\left(n_2 - n_{x_2}\right)^2 + E_{\mathrm{I}}\left(n_1 - n_{x_1}\right)\left(n_2 - n_{x_2}\right)$$

$$n_{x1} = V_{\mathrm{g}1}\frac{C_{\mathrm{g}}}{e}$$

$$n_{x2} = V_{\mathrm{g}2}\frac{C_{\mathrm{g}}}{e}$$

*inter-island interaction energy controlled by* $C_{\mathrm{I}}$

* Electrostatic energy change for transitions in lower island

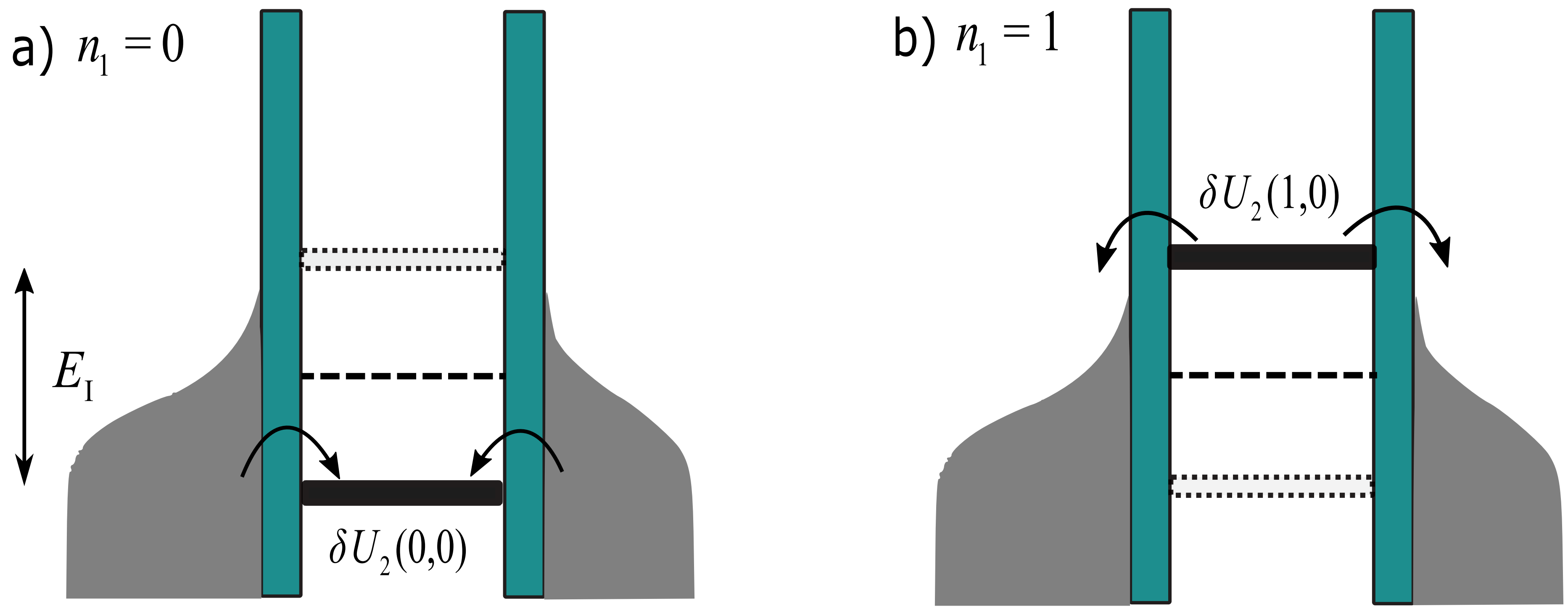
$$\delta U_2(n_1,n_2) = U(n_1,n_2+1) - U(n_1,n_2)$$

*depends on the charge state in island 1*

# Capacitively-coupled metallic islands

- Dragged heat current results from **energy transferred** from drive circuit, through this mechanism

upper island is empty | upper island is occupied




$$n_{x1} = n_{x2} = \frac{1}{2}$$

$$\delta U_2(0,0) = E_C(1 - 2n_{x2}) - E_\mathrm{I} n_{x1},$$

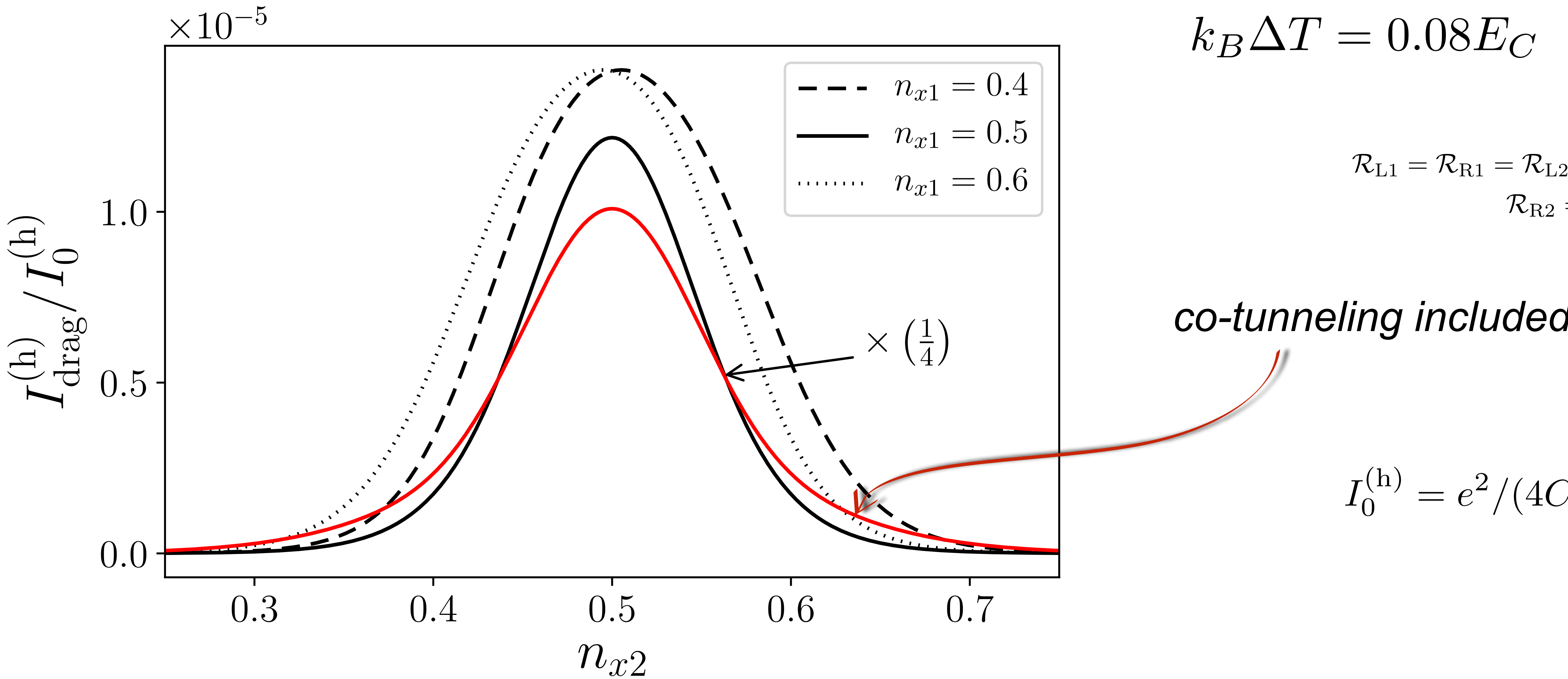
$$\delta U_2(1,0) = E_C(1 - 2n_{x2}) + E_\mathrm{I}(1 - n_{x1}),$$

- Heat current associated to this processes: $E_\mathrm{I}$
- Heat currents are modulated through gate voltages

# Capacitively-coupled metallic islands

➢ Dependence on gate voltage

$$E_{\mathrm{I}} = 0.4E_C$$
$$k_B T = 0.05E_C$$
$$k_B \Delta T = 0.08E_C$$




$\mathcal{R}_{\mathrm{L1}} = \mathcal{R}_{\mathrm{R1}} = \mathcal{R}_{\mathrm{L2}} = 5R_Q$
$\mathcal{R}_{\mathrm{R2}} = 10R_Q$

$I_0^{(\mathrm{h})} = e^2/(4C^2\mathcal{R})$

☑ Width independent of $E_I$, but controlled by temperature
☑ Maximum occurs in the symmetric energy configuration

# Capacitively-coupled metallic islands

CNRNANO PISA NEST

## ➢ Sequential tunnelling, small bias

$$n_{x1} = n_{x2} = \frac{1}{2}$$

expansion in $\Delta T/T$

$$I^{(\mathrm{h})}_{\mathrm{drag}} = \frac{\xi \mathcal{R}_{\parallel}}{6e^2\mathcal{R}}\left[\frac{1}{\mathcal{R}_{\mathrm{L2}}} - \frac{1}{\mathcal{R}_{\mathrm{R2}}}\right] \mathrm{csch}\xi \left[2\xi\left(\frac{\pi^2}{4} + \xi^2\right)\mathrm{csch}\xi - \left(\frac{\pi^2}{2} + 3\xi^2\right)\mathrm{sech}\xi\right](k_B \Delta T)^2$$

☑ Second order in $\Delta T$

$$\xi = \frac{E_{\mathrm{I}}}{4k_B T}$$

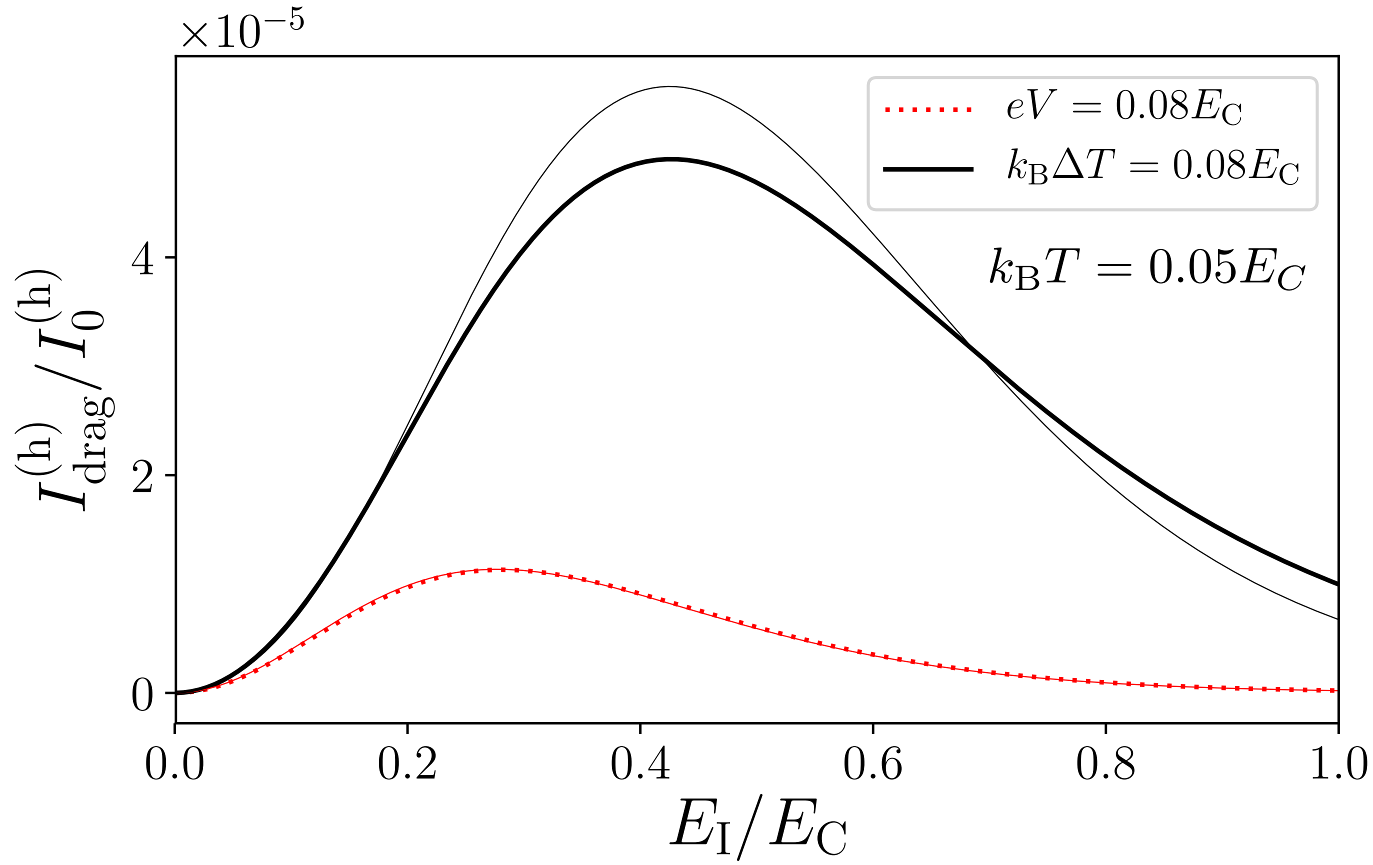
expansion in $eV/E_C$

$$I^{(\mathrm{h})}_{\mathrm{drag}} = \frac{\xi \mathcal{R}_{\parallel}}{16\mathcal{R}}\left[\frac{1}{\mathcal{R}_{\mathrm{L2}}} - \frac{1}{\mathcal{R}_{\mathrm{R2}}}\right] \mathrm{csch}\xi \left[\xi \mathrm{csch}\xi - \mathrm{sech}\xi\right] V^2$$

☑ Second order in $V$

# Capacitively-coupled metallic islands

## ➢ Sequential tunnelling, dependence on inter-island coupling




*thin curves ⇢ analytical*
***thick** curves ⇢ numerical*

$$I_0^{(\mathrm{h})} = e^2/(4C^2\mathcal{R})$$

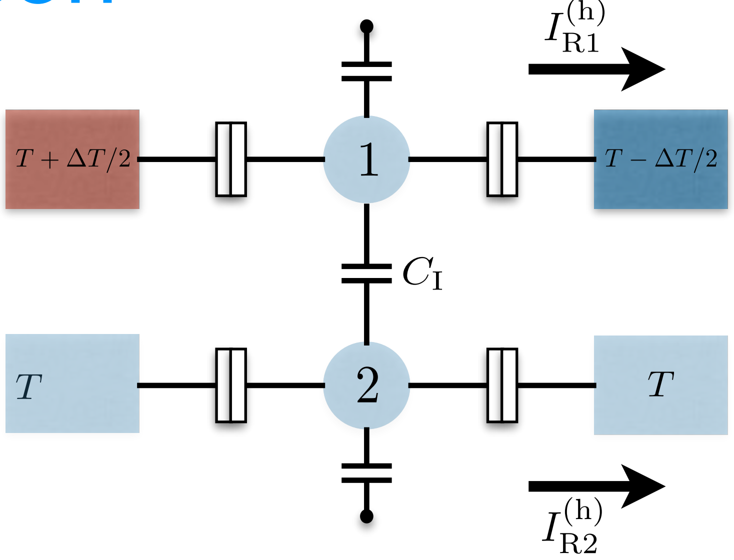
- ☑ Maximal for intermediate values
- ☑ Position of the maximum obtained from analytics

$$E_\mathrm{I}^\mathrm{max} \simeq 8.5 k_B T$$
$$E_\mathrm{I}^\mathrm{max} \simeq 5.5 k_B T$$

- ☑ Deviation due to *T* not small

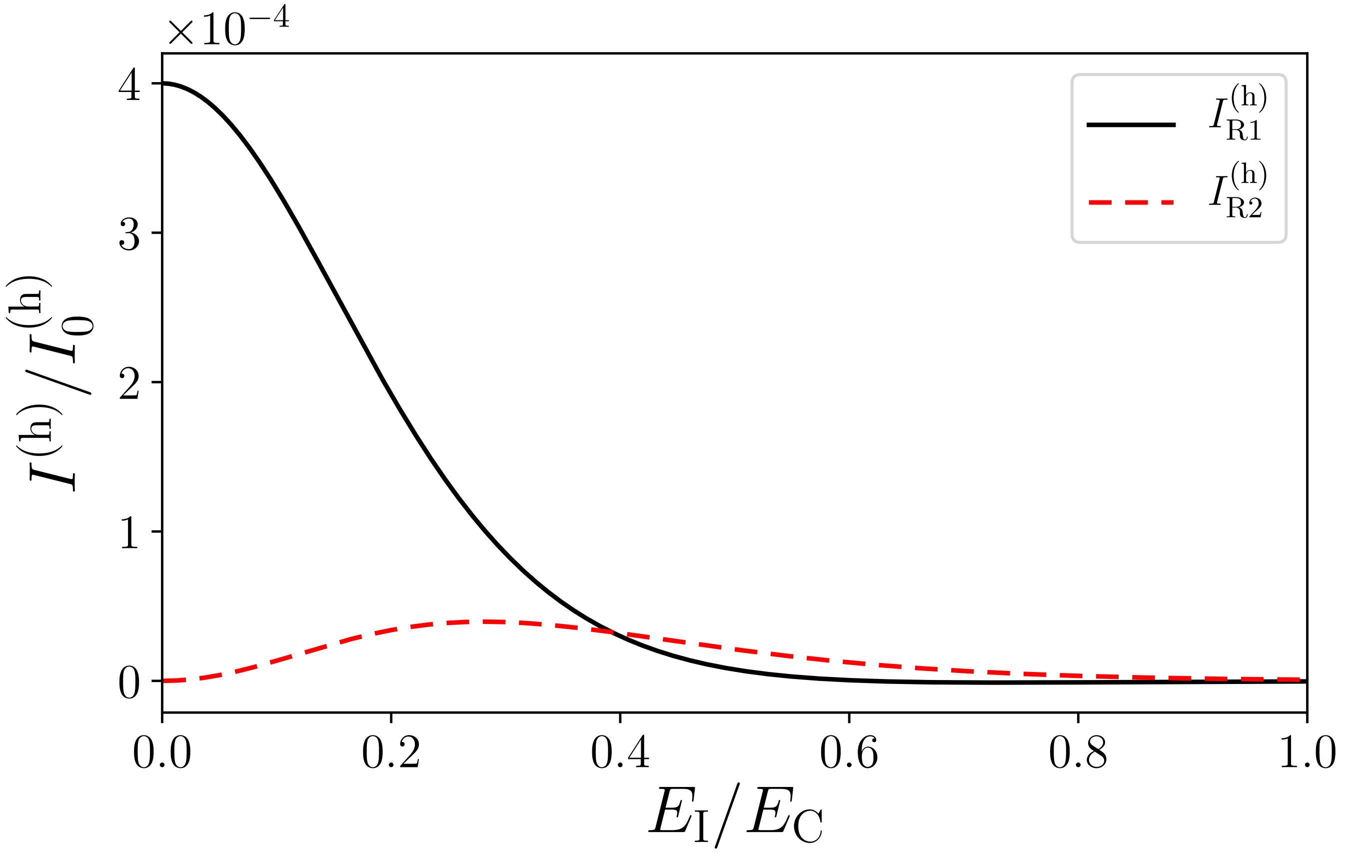
# Capacitively-coupled metallic islands

## ➢ Sequential tunnelling, drag-drive comparison




- ☑ Temperature-biased case: $I_{\rm R2}^{\rm (h)} < I_{\rm R1}^{\rm (h)}$

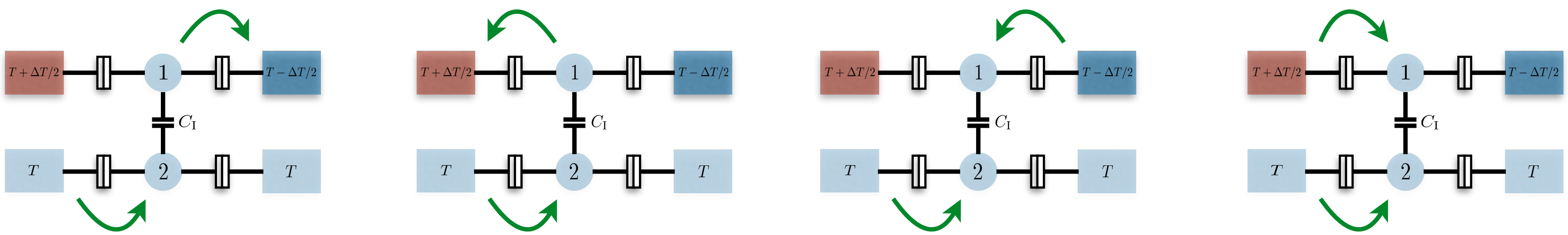
- ☑ Voltage-biased case: $I_{\rm R2}^{\rm (h)} > I_{\rm R1}^{\rm (h)}$

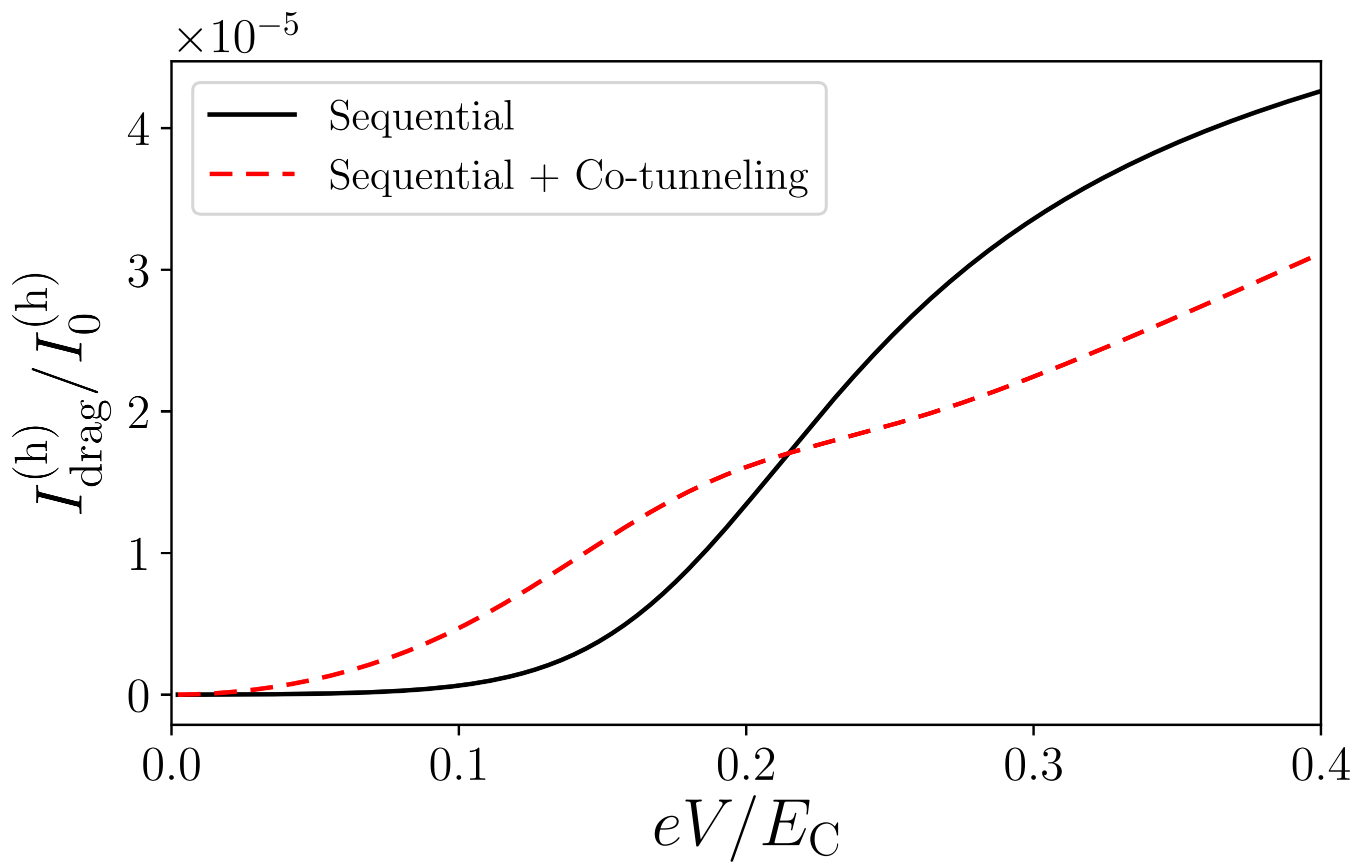



- ☑ Larger heat current in the drag circuit for large interaction

# Capacitively-coupled metallic islands

- Cotunnelling contributions, large bias



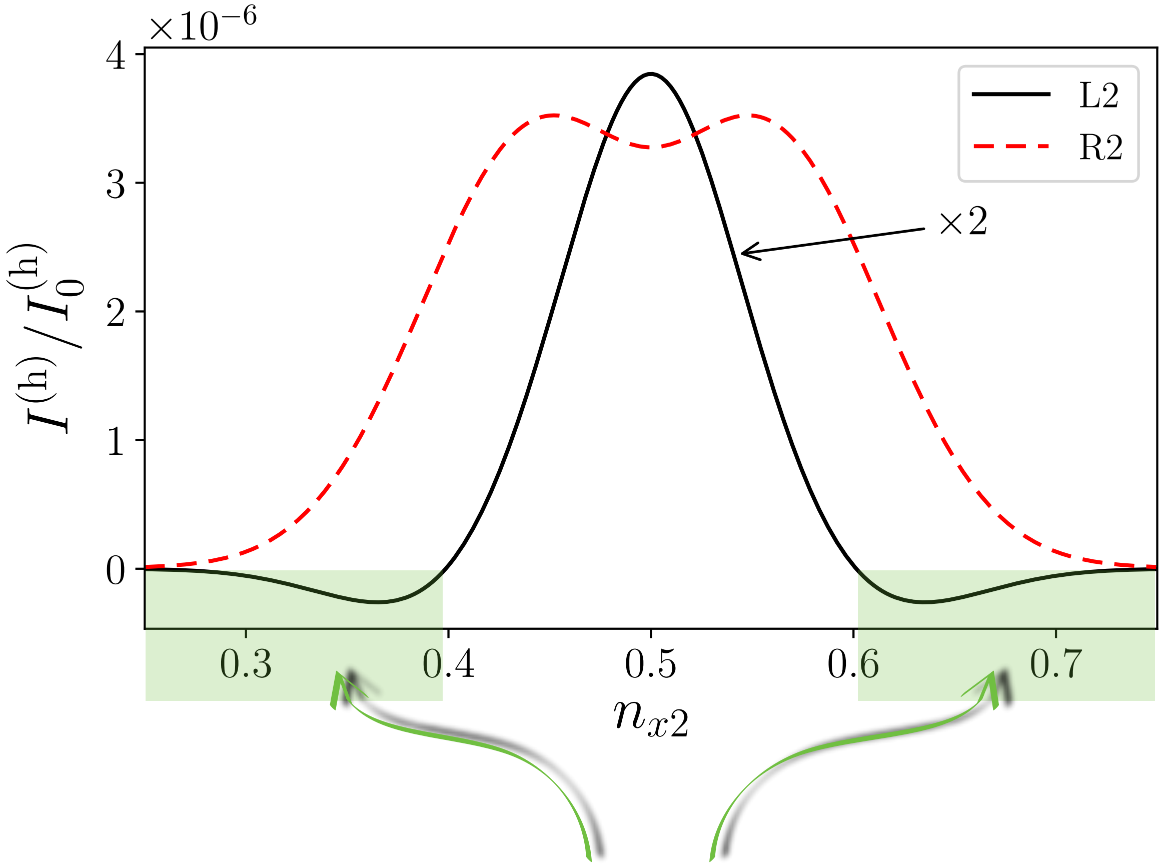



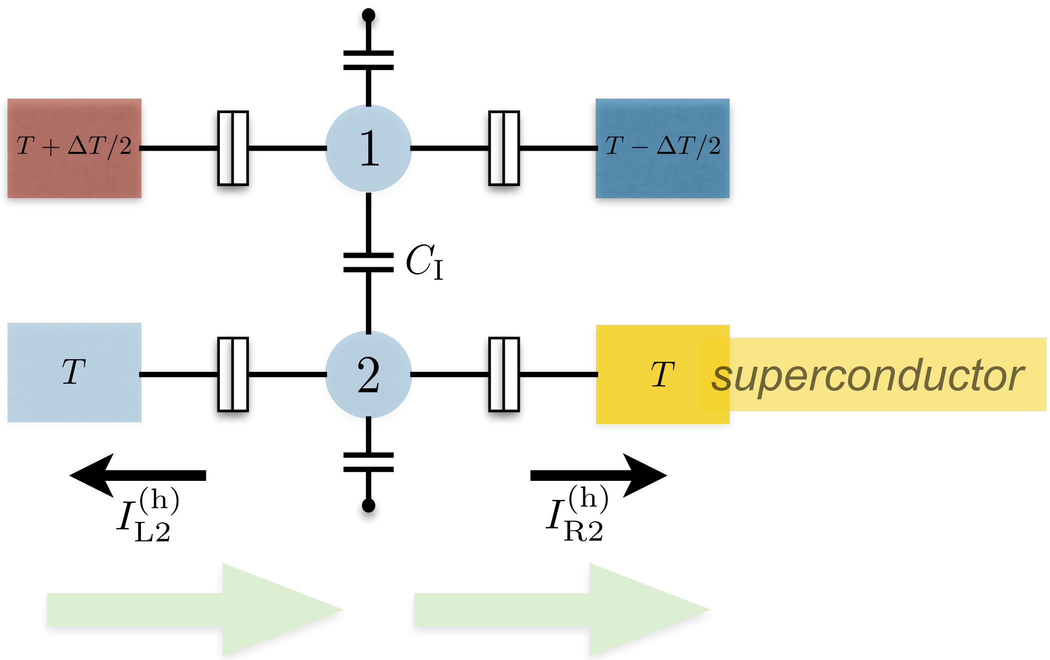

☑ Non-trivial contributions
☑ Quadratic dependence on *V* and $\Delta T$

# Capacitively-coupled metallic islands

## ➢ Energy-dependent couplings: superconductor

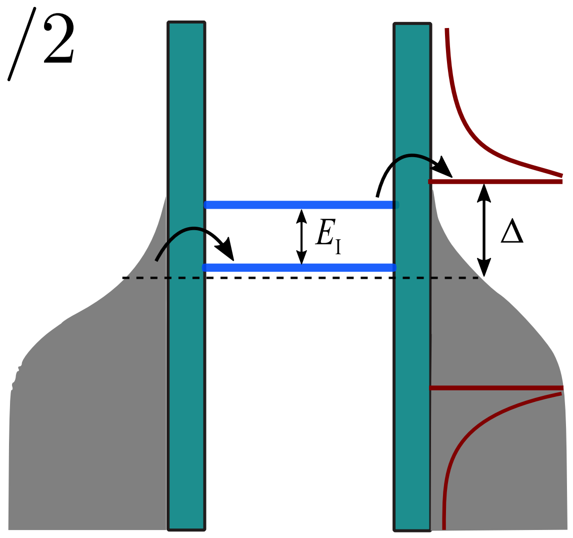
$\Delta \ll E_C$



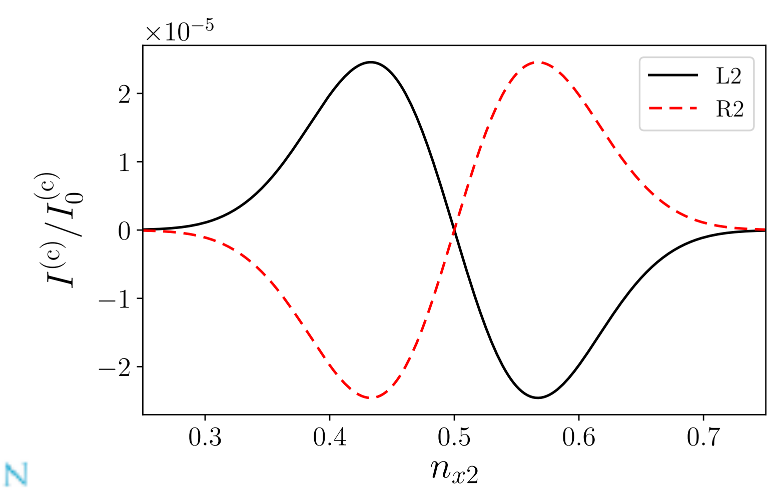




*heat dragged to the right*

$n_{x2} < 1/2$




☑ Finite dragged charge



# Conclusions

➢ Thermal drag in capacitively-coupled metallic islands

- ✱ Sequential tunneling regime
- ✱ Co-tunneling contributions

- ✱ Analytic expressions for heat currents for small biases

- ✱ Dependence on the inter-islands coupling

- ✱ Energy-Dependent island-lead couplings