

MANY PARTICLE SIGNATURE OF A MOBILITY EDGE IN A BICHROMATIC LATTICE

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Jacopo Settino

Frank Hekking Memorial Workshop, Les Houches, 28 Jan 2018

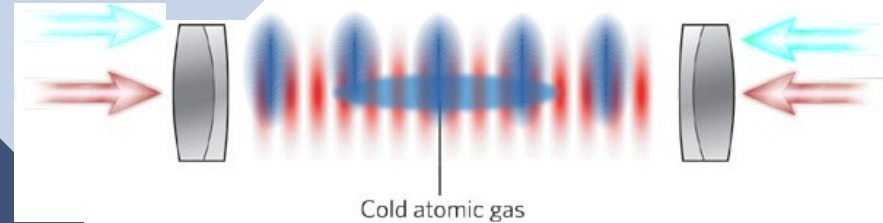
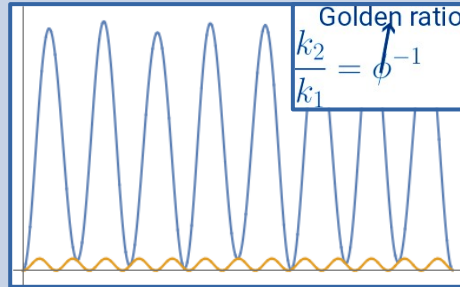
1

Incommensurability

What is a bichromatic lattice?

$$V(x) = V_1 \sin^2(k_1 x) + V_2 \sin^2(k_2 x + \phi)$$

1



Incommensurability

What is a bichromatic lattice?

Phase Transition

Tight Binding Approximation ($V_1 \gg E_r$)

MAPPING

ANDRÉ-AUBRY MODEL

$$\hat{H} = \Delta \sum_j \cos(2\pi\tau j) |j\rangle\langle j| - J \sum_j (|j+1\rangle\langle j| + |j\rangle\langle j+1|)$$

Disorder

Wannier states

Hopping

Phase Transition

Tight Binding Approximation ($V_1 \gg E_r$)

MAPPING

ANDRÉ-AUBRY MODEL

$$\hat{H} = \Delta \sum_j \cos(2\pi\tau j) |j\rangle\langle j| - J \sum_j (|j+1\rangle\langle j| + |j\rangle\langle j+1|)$$

Incommensurability

Wannier states

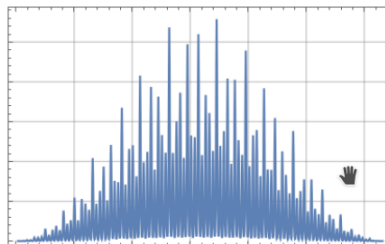
Hopping

Phase Transition

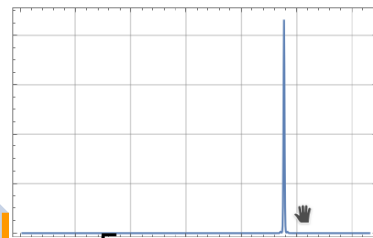
Metal to Insulator

$$2J = \Delta$$

Delocalized



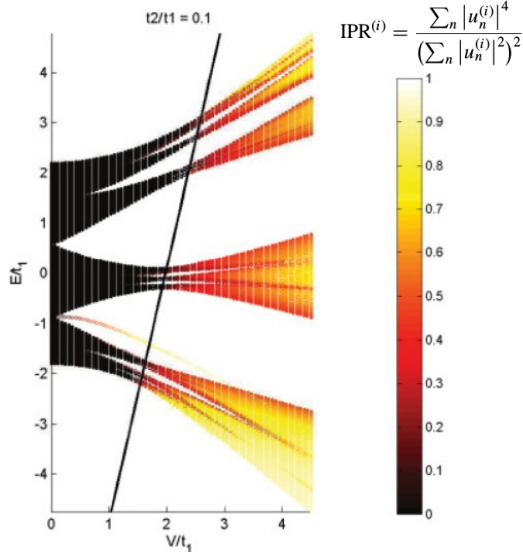
Localized



Mobility Edge

ANDRÉ-AUBRY NNN MODEL

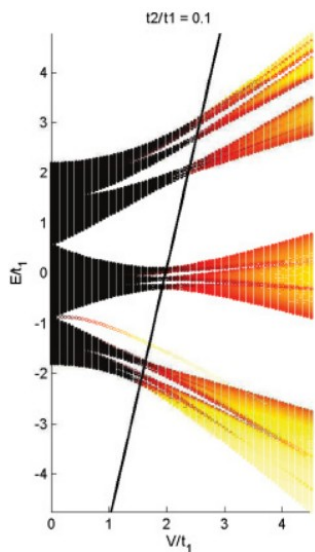
$$\left(t_2(u_{n+2} + u_{n-2}) + t_1(u_{n+1} + u_{n-1}) + V \cos(2\pi\alpha n + \delta) u_n \right) = E u_n$$



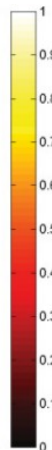
Mobility Edge

ANDRÉ-AUBRY NNN MODEL

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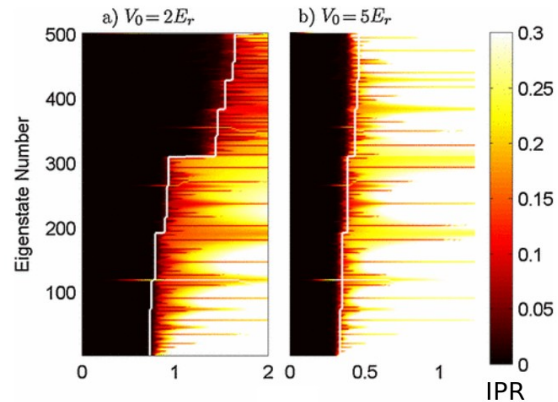
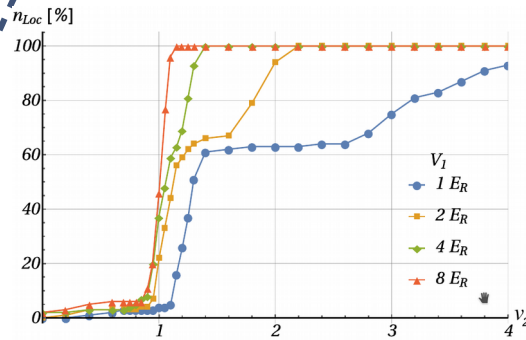


$$IPR^{(i)} = \frac{\sum_n |u_n^{(i)}|^4}{(\sum_n |u_n^{(i)}|^2)^2}$$



CONTINUOUS MODEL

$$V(x) = V_1 \sin^2(k_1 x) + V_2 \sin^2(k_2 x + \phi)$$



*“Is it possible to observe the presence of the **mobility edge** by looking at **many-body measurable quantities**?”*

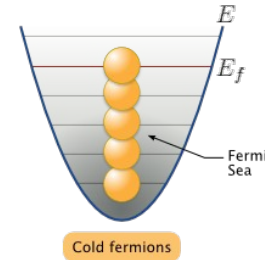
2

Many-body systems

- Noninteracting fermions
- Hardcore bosons

Non-interacting fermions

$T=0$



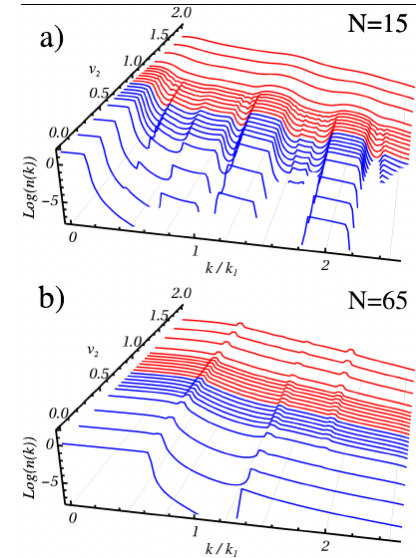
Reduced Single Particle Density Matrix

$$\rho_F(x, y) = \int dx_2 \dots dx_N \Psi_F^*(x, \dots, x_N) \Psi_F(y, \dots, x_N)$$



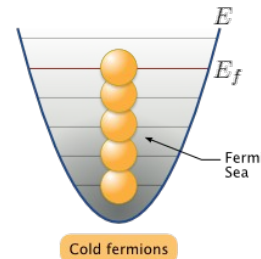
Momentum distribution

$$n_F(k) = \frac{1}{2\pi} \int dx dy \exp^{ik(x-y)} \rho_F(x, y)$$



Non-interacting fermions

$T=0$



Reduced Single Particle Density Matrix

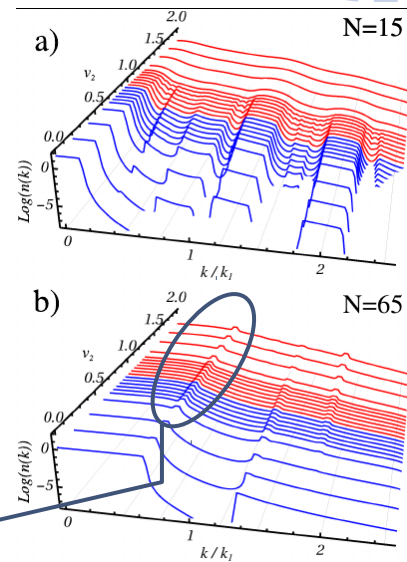
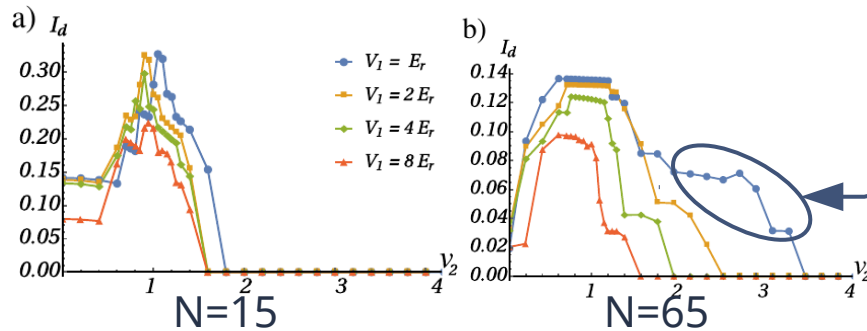
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Momentum distribution

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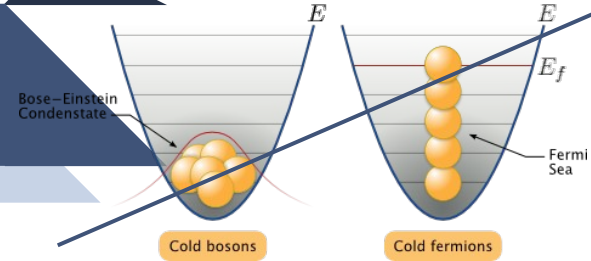
Intensity of peaks



Hardcore Bosons

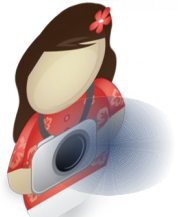
Strongly interacting

$T=0$

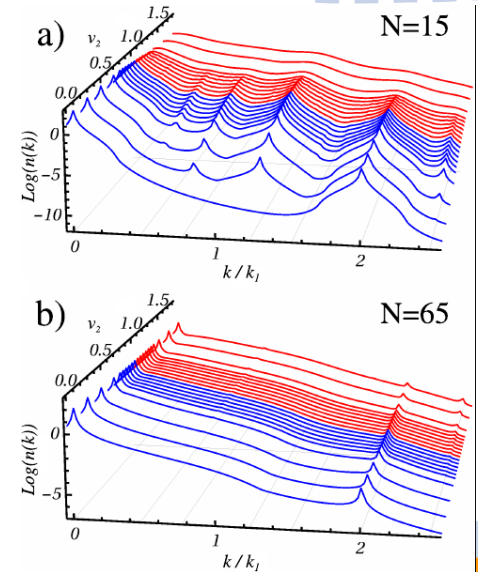


$$U_{ij} = g \delta(x_i - x_j) \quad g \rightarrow \infty$$

$$\Psi_B = A \Psi_F \quad A = \prod_{1 \leq i < j \leq N} \text{sgn}(x_i - x_j)$$



Momentum distribution

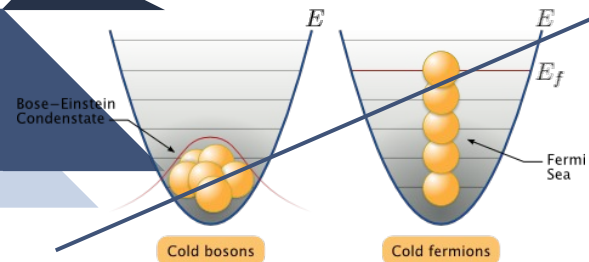


Hardcore Bosons

Strogly interacting

$$U_{ij} = g \delta(x_i - x_j) \quad g \rightarrow \infty$$

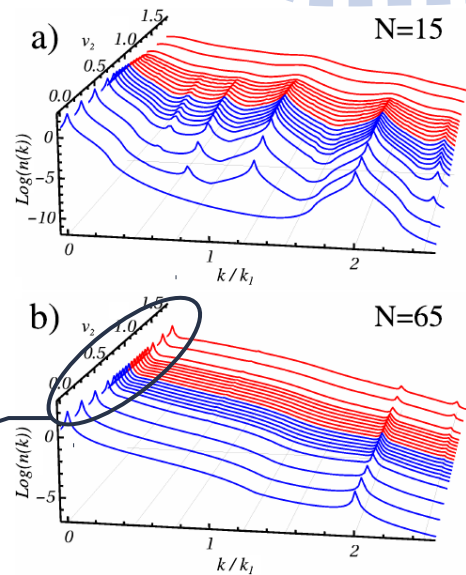
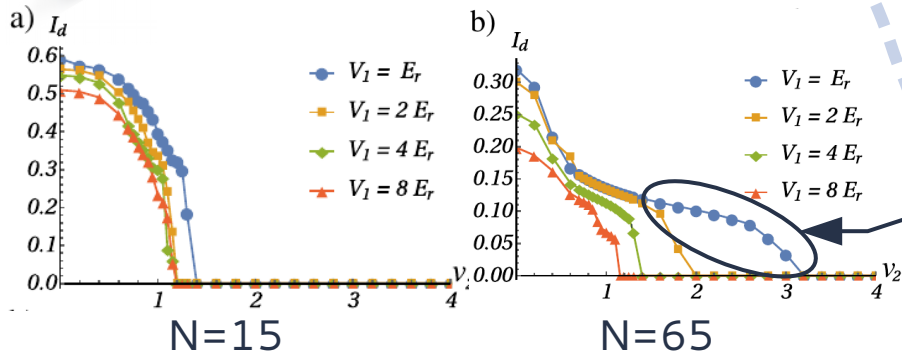
$T=0$



$$\Psi_B = A \Psi_F \quad A = \prod_{1 \leq i < j \leq N} \text{sgn}(x_i - x_j)$$




Momentum distribution
Intensity of peaks





THANK YOU!



Signatures of the single-particle mobility edge in the ground-state properties of Tonks-Girardeau and noninteracting Fermi gases in a bichromatic potential

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We explore the ground-state properties of cold atomic gases focusing on the cases of noninteracting fermions and hard-core (Tonks-Girardeau) bosons, trapped by the combination of two potentials (bichromatic lattice) with incommensurate periods. In the tight-binding limit, the single-particle states in the lowest occupied band show a localization transition and, as the strength of the second potential is increased above a certain threshold when the tight-binding approximation does not hold, a mobility edge is found. Here, we study how the crossover from the discrete to the continuum behavior occurs, and prove that signatures of the localization transition and mobility edge clearly appear in the generic many-body properties of the systems.

THE MODEL

We will consider an external potential which describes a 1D bichromatic lattice. The ratio between the two oscillating terms is chosen to be the Golden Ratio.

$$V(x) = V_1 \sin^2(k_1 x) + V_2 \sin^2(k_2 x) \quad \frac{k_2}{k_1} = \text{Golden Ratio}$$


$V_1 \gg V_2 = \frac{h^2 \omega}{2m}$ (Tight Binding approximation)

Exponentially localized


THE APPROXIMATION

If the main potential is much larger than the second energy, the single particle problem can be mapped into the so called **André-Aubry model**^[1]

$$\hat{H} = \Delta \sum_j \cos(2\pi\alpha j) [c_j^\dagger - c_j] \sum_l [l + 1/2] [c_l^\dagger + c_l]$$

Incommensurability $\alpha = \frac{V_2}{V_1 + V_2}$ **hopping**

At $\Delta \ll V_1$ there is a transition to insulator phase transition, with corresponding delocalized or exponentially localized eigenstates.



Delocalized **Exponentially localized**

SINGLE PARTICLE MOBILITY EDGE

When the TB approximation doesn't hold ($V_1 \sim 50V_2$) a mobility edge (ME) appears such that, for a fixed value of V_2 , states with energy lower than the ME are localized while the others are delocalized. The ME is found at higher energies for increasing V_2 .

Fig. Number of localized states as a function of V_2 for different values of V_1 . An approximation is considered to be localized if the IPR is greater than $1/(S-1)$, where l is the distance between two neighboring sites.


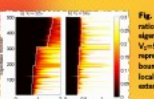


Fig. Inverse participation ratio of the first band eigenstates for $V_1=50V_2$ and $V_1=10V_2$. The solid curves represent the analytical boundary between spatially localized and spatially extended states l .



"Is it possible to observe the presence of the mobility-edge by looking at many-body measurable quantities?"

NON INTERACTING FERMIONS

We consider the effect of the ME of the single-particle spectrum on the ground-state properties of a system of N noninteracting fermions. We focus on the reduced single-particle density matrix (RSPDM)

$$\rho_{ij}(x, y) = \int dx_1 \dots dx_N \Psi_{i,j}^*(x_1, \dots, x_N) \Psi_j(x_1, \dots, x_N)$$

Its Fourier transform gives the momentum distribution (MD)

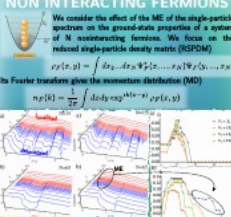
$$n_{ij}(k) = \frac{1}{N} \int dx dy \exp(ik(x-y)) \rho_{ij}(x, y)$$


Fig. The MD offers a signature of the localization transition and, for $V_1=50V_2$ and $V_1=10V_2$, persistent structures beyond the transition point manifest the presence of the Mobility Edge.

TONKS-GIRARDEAU GAS

HARD-CORE BOSONS

We look at a strongly interacting boson gas ($\{c_i, c_i^\dagger, c_i^2 = 0, \dots\}$) with $\beta \rightarrow \infty$ (i.e., the Tonks-Girardeau gas). The strong repulsive force allows us to map the system into an ideal spinless fermionic one. We focus again on the RSPDM and the MD. We have evaluated area under the edge peaks of the MD, in order to identify the ME transition and the presence of the ME, and the entropy of the RSPDM.

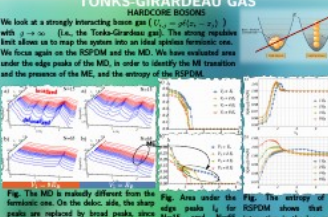


Fig. The MD is radically different from the fermionic one. On the delocalized side, the sharp peaks are replaced by broad peaks, since bosons tends to occupy the lowest energy levels, but the interaction broadens them. On the localized side, the MD features an in the fermionic case. Peaks persistence in the localized region witnesses the ME presence.

Fig. Area under the edge peaks l_i for different values of V_1 .

Fig. Area under the edge peaks l_i for different values of V_1 .

Fig. The entropy of the RSPDM shows that interactions in the 1D regime makes bosons occupy one effective SP state each in the delocalized side; the increasing of structures (V_1 or v_1) increases the number of effective occupied SP states.

REFERENCES [1] M. Modugno, New Journal of Physics, 11 (2009) 033023 [2] J. Bidin, S. Das Sarma Physical Review Letters 104, 076602 (2010)

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