

# Yu-Shiba-Rusinov bands in superconductors in contact with a magnetic insulator

Wolfgang Belzig (University of Konstanz)

Detlef Beckmann (KIT)

J. Magn. Magn. Mater. (in press) [arXiv:1710.04413]

Frank Hekking Memorial Workshop, Les Houches 2018

Institut für Theoretische Festkörperphysik  
Prof. G. Schön, F.W.J. Hekking (10/22, Tel.: 608-3365) , A. Odintsov

**Hauptseminar: Theorie der Supraleitung**  
(Besprechung am 26. Oktober)

(1) Meissner-Effekt. Befindet sich ein Metall in einem Magnetfeld  $H < H_c$ , so wird das Feld, nachdem der Übergang zum supraleitenden Zustand stattgefunden hat, aus dem Supraleiter verdrängt. Das Feld ist jedoch nur im Inneren einer massiven Probe null: es fällt ab innerhalb einer dünnen Schicht an der Oberfläche des Supraleiters. Die Dicke dieser Schicht wird Eindringtiefe genannt. Innerhalb der Eindringsschicht fließt ein Dauerstrom, der im Inneren ein Magnetfeld erzeugt, daß das äußere Feld kompensiert.

(a) Wir betrachten zuerst einen idealen Leiter im normalen Zustand. Geben Sie im stationären Fall das  $\vec{E}$ -Feld im Leiter an. Zeigen Sie somit, daß innerhalb des Leiters  $\partial\vec{B}/\partial t = 0$  gilt. Was passiert, wenn ein Magnetfeld angeschaltet wird? Warum ist dies kein Meissner-Effekt?

(b) Jetzt betrachten wir ein nicht-ideales Metall. Die Bewegung der Elektronen in einem elektrischen Feld  $\vec{E}(\vec{r}, t)$  wird klassisch durch die Bewegungsgleichung

$$m \frac{\partial \vec{v}}{\partial t} + \frac{m}{\tau} \vec{v} = e \vec{E} \quad (1)$$

beschrieben. Die Stoßzeit  $\tau$  berücksichtigt die Streuung an Verunreinigungen. Der Strom sei  $\vec{j} = ne\vec{v}$  ( $n$  ist die Dichte der Elektronen). Unter welcher Bedingung ergibt sich jetzt die erste London-Gleichung (Gleichung (1-3) in Tinkham)?

(c) Mit Hilfe dieser Gleichung und der Phänomenologie des Meissner-Effektes zeige man, daß

$$\text{rot } \Lambda \vec{j} + c^{-1} \vec{H} = 0$$

im Inneren einer supraleitenden Probe gilt. Zeigen Sie somit, daß  $\Delta \vec{H} = \lambda_L^{-2} \vec{H}$  ist, und geben Sie einen Ausdruck für die Eindringtiefe  $\lambda_L$  an.

[Hinweis: Benutzen Sie die Maxwell-Gleichungen in der quasistationären Näherung. Das Metall habe die Materialgleichungen  $\vec{D} = \epsilon \vec{E}$ ,  $\vec{B} = \mu \vec{H}$ .]

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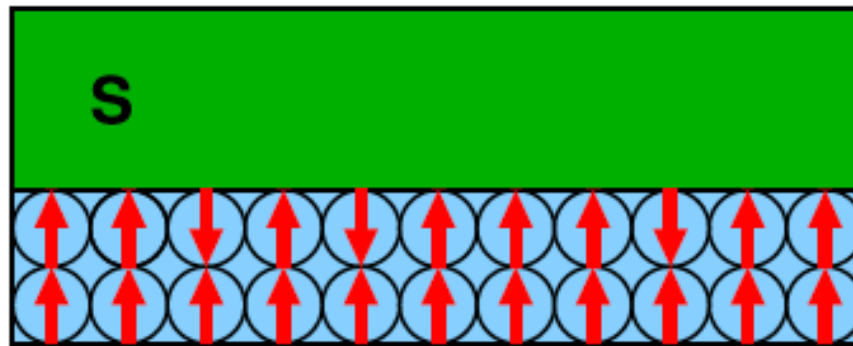
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## Outline

- Impurity effects (elastic, magnetic) in superconductors
- Boundary effects on superconductors
- Strong spin-dependent boundaries and Shiba states
- Hybridization effects of multiple Shiba bands

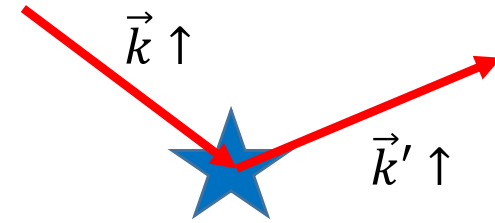


## Impurity effects in superconductors: elastic scattering

Quasiclassical Eilenberger equation:

$$-i\vec{v}\vec{\nabla}\check{g} = [E\hat{t}_3 + \hat{\Delta} + \check{\Sigma}, \check{g}]$$

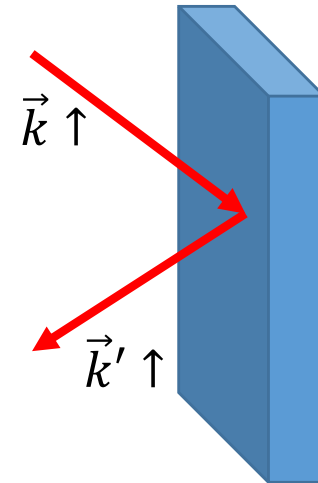
Keldysh-Nambu-Spin-Green function



Selfenergy due to elastic scattering:  $\check{\Sigma}_{el} = \frac{i}{2\tau_{el}} \langle \check{g} \rangle_{\vec{v}_F}$

Homogeneous situation:  $\check{g} = \langle \check{g} \rangle_{\vec{v}_F}$

$$[\check{\Sigma}_{el}, \check{g}] = 0$$

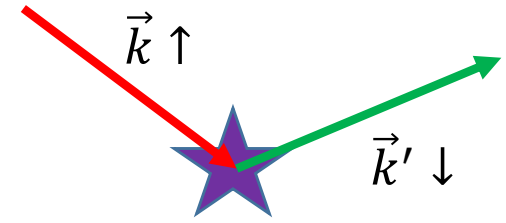


Elastic scattering does not affect the thermodynamic properties (e.g. the spectrum) → Anderson theorem (1959)

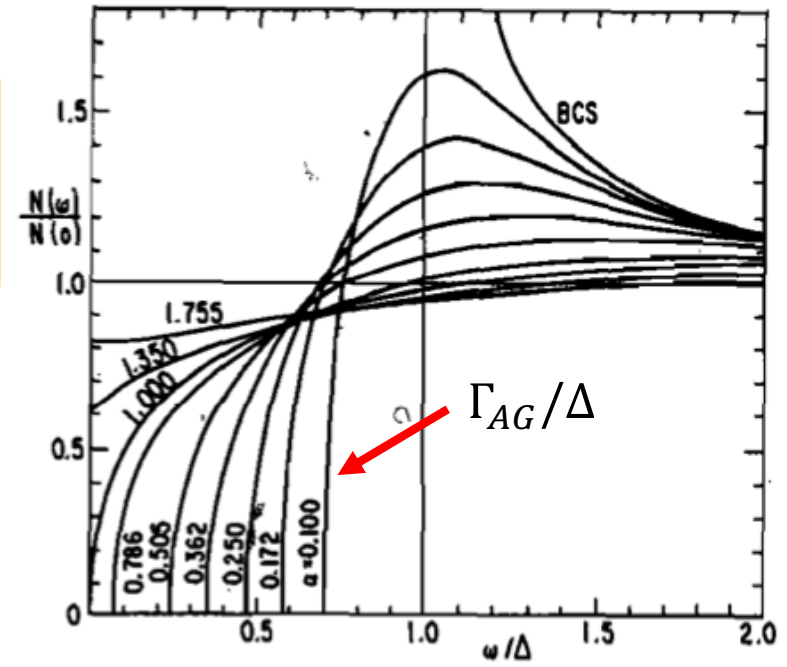
# Impurity effects in superconductors: magnetic scattering

Self energy due to spin flip:  $\check{\Sigma}_{sf} = \frac{i}{2} \Gamma_{AG} \check{k}_z \langle \check{g} \rangle_{\vec{v}_F} \check{k}_z$

$$[\check{\Sigma}_{sf}, \langle \check{g} \rangle_{\vec{v}_F}] \neq 0$$



Magnetic impurities suppress superconductivity [Abrikosov, Gorkov 1961]  
 → Gapless superconductivity



[Ambegaokar, Griffin (1965), Skalski (1965), Maki in Parks (1969)]

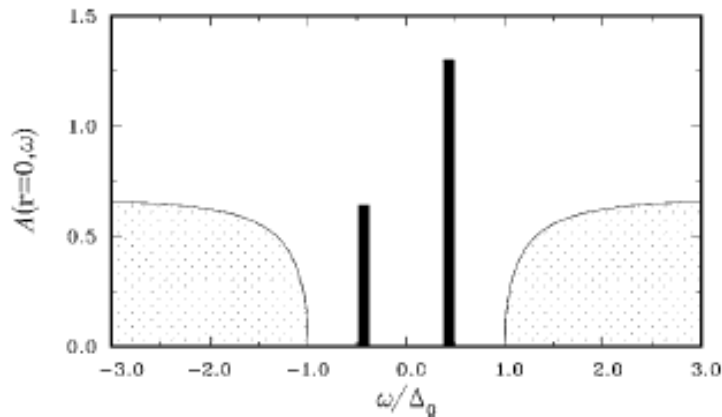


# Impurity scattering: From Yu-Shiba-Rusinov states to Shiba bands

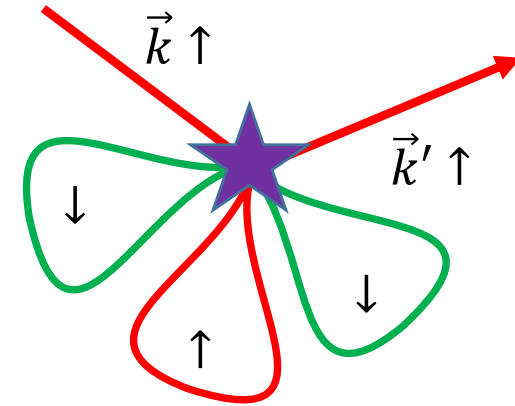
$$H_{int} = J\vec{S}\hat{S} \quad \gamma = \pi J S N_0 / 2$$

[Yu (1965); Shiba (1965); Rusinov (1965)]

Single impurity: bound state energy  $\epsilon_B = \frac{1-\gamma^2}{1+\gamma^2}$

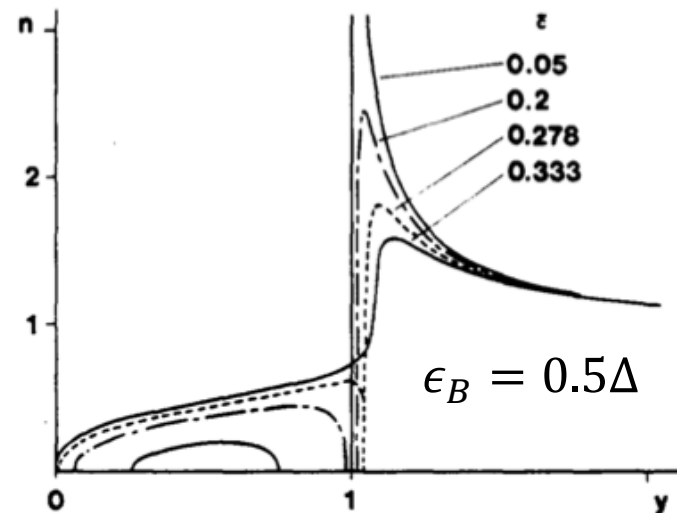


[From Balatski, Vekhter, Zhu (2006)]

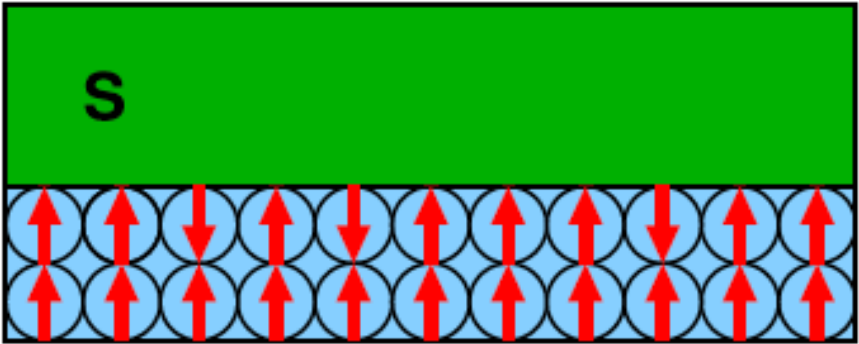


Many impurities:  
formation of an impurity band  $\hat{\Sigma} = \epsilon \hat{t}$

$$\begin{aligned} \bar{t} &= \gamma^2 \hat{k}_z \hat{g} \hat{k}_z + \gamma^4 \hat{k}_z \hat{g} \hat{k}_z \hat{g} \hat{k}_z \hat{g} \hat{k}_z + \dots \\ &= \frac{\gamma^2 \hat{k}_z \hat{g} \hat{k}_z}{1 - \gamma^2 (\hat{k}_z \hat{g})^2} \end{aligned}$$



[Zittartz, Bringer, Müller-Hartmann (1972)]



## Boundary conditions for quasiclassical Greens functions

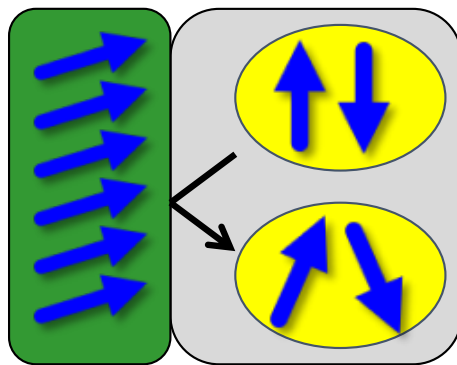
Spin-independent scattering:

- General non-linear [Zaitsev 1984]
- Small transmission, diffusive [Kupriyanov, Lukichev 1988]
- Arbitrary transmission, diffusive [Nazarov (1999)]

Spin-dependent scattering:

- General non-linear, implicit [Rainer, Sauls, Millis (1988)]
- Weak spin-dependence, diffusive [Huertas-Hernando, Nazarov, WB (2002)]
- Weak spin-dependence<sup>2</sup> [Cottet, Huertas-Hernando, WB, Nazarov (2009)]
- Strong spin-dependence, monodomain [Machon, WB (2015)]
- Strong spin-dependence, arbitrary domain [Eschrig et al. (2015)]

### Spin mixing



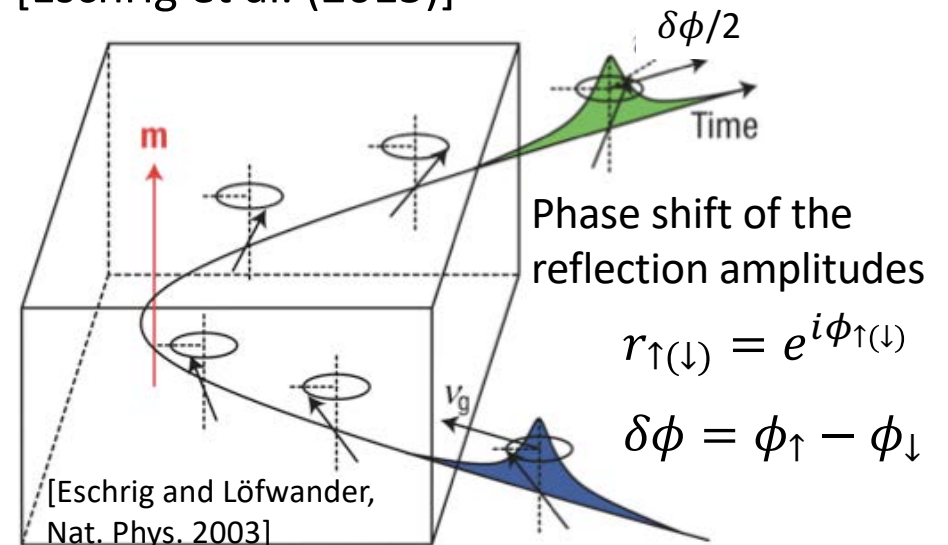
$$G^\phi = G_Q \sum_n \delta\phi_n$$

- 100% spin valve
- Spin triplet pairing
- Spin supercurrent

[Huertas-Hernando, Nazarov, Belzig PRL 2002]

Consequence: expansion in the spin-mixing angle (for insulating interfaces)

$$\hat{\Sigma}_{int} = -i\hat{k}_z \frac{G}{G_Q} E_{Th} \langle \delta\phi \rangle_{\vec{v}_F} + \frac{G}{G_Q} E_{Th} \langle \delta\phi^2 \rangle_{\vec{v}_F} \hat{k}_z \hat{g} \hat{k}_z + \dots$$



$$r_{\uparrow(\downarrow)} = e^{i\phi_{\uparrow(\downarrow)}}$$

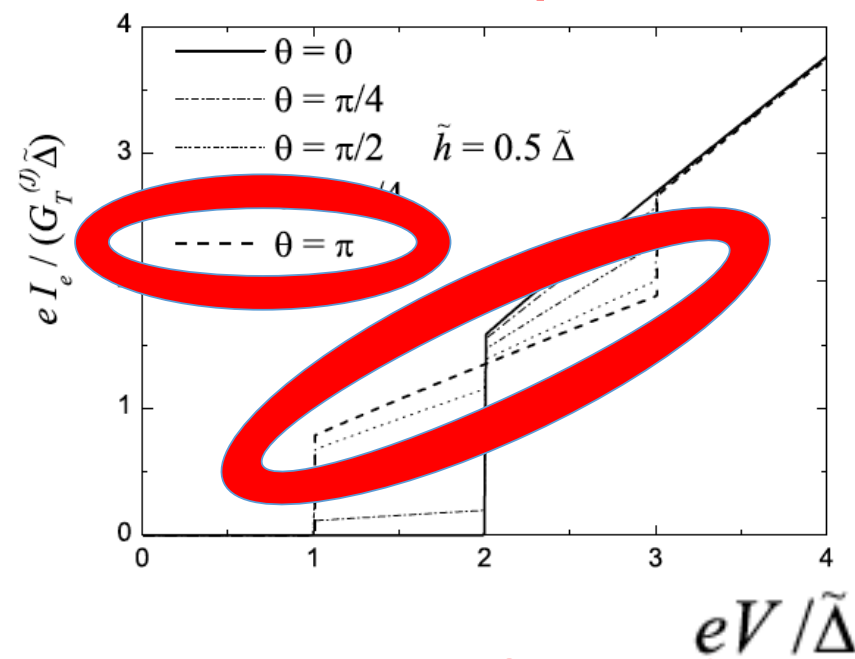
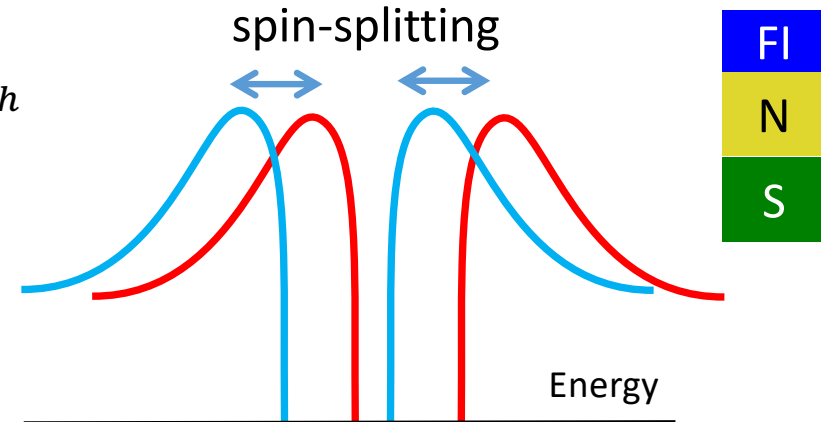
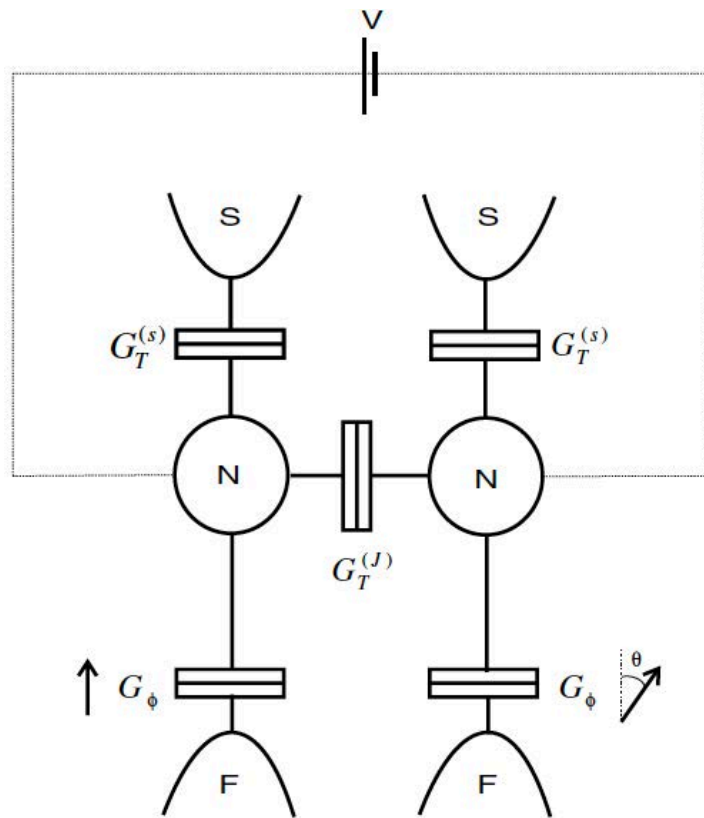
$$\delta\phi = \phi_{\uparrow} - \phi_{\downarrow}$$

# Basis of „absolute spin-valve effect“:

## Absolute Spin-Valve Effect with Superconducting Proximity Structures

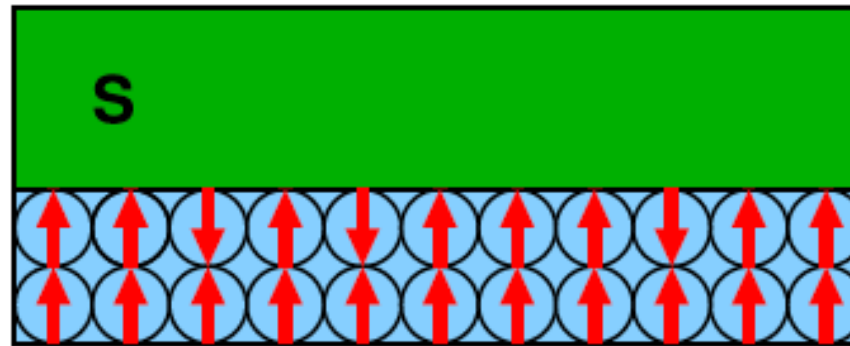
D. Huertas-Hernando, Y. V. Nazarov, and W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002).

$$\hat{\Sigma}_{int} = -i\hat{k}_z \frac{G}{G_Q} E_{Th} \langle \delta\phi \rangle \vec{v}_F = -i\hat{k}_z \frac{G_\phi}{G_Q} E_{Th}$$

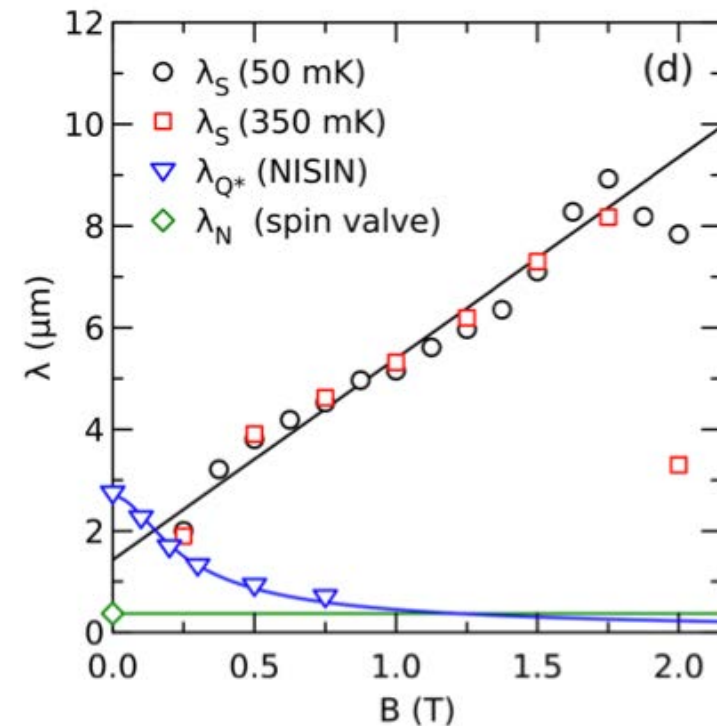
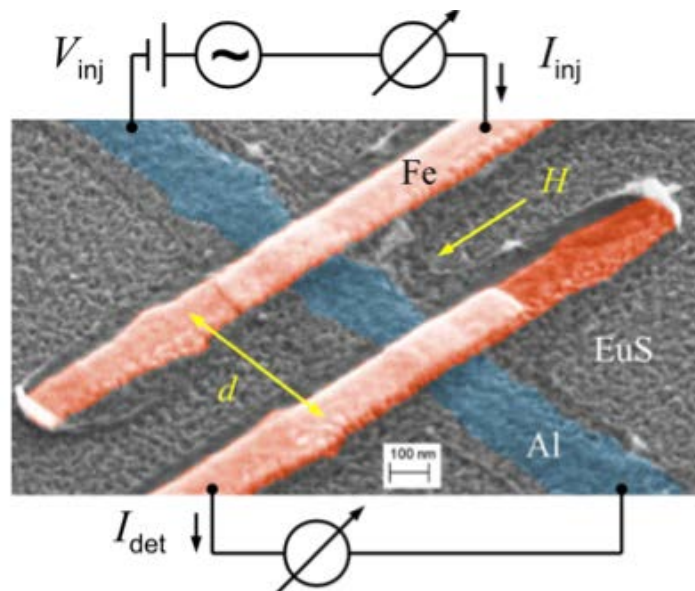


**100% spin-polarized current!**

Testing the boundary:  
a superconductor in contact  
with a magnetic insulator



**Motivation:** Experiments on spin transport



Spin-relaxation length  $\sim 10\mu\text{m}$

[Hübler, Wolf, Beckmann, von Lohneysen (2012) Wolf, Sürgers, Fischer, Beckmann (2014)]

From Eilenberger/Usadel/Circuit theory we obtain:

[Machon, Belzig, 2015]

$$-iEf_\sigma - \Delta g_\sigma = 2if_\sigma \frac{E_{Th}}{N} \sum_n \frac{\sigma \sin\left(\frac{\delta\phi_n}{2}\right)}{\cos\left(\frac{\delta\phi_n}{2}\right) - i\sigma g_\sigma \sin\left(\frac{\delta\phi_n}{2}\right)}$$

Sum over all spin-active channels  
 → Fraction  $r_N$  of all channels

$\delta\phi_n$ : spin-dependent phase shifts

Effective exchange parameter:  $r_N E_{Th} = \varepsilon$

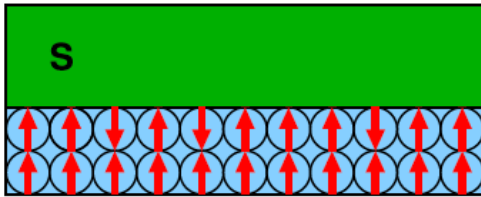
Mapping: selfenergy for unpolarized interface spins can be related to impurity selfenergy due to strong magnetic scatterers (Shiba states → Shiba impurity band)

$$\sum_\sigma [r.h.s](\sigma) = [\hat{\Sigma}_{int}, \hat{g}] \quad \text{with} \quad \hat{\Sigma}_{int} = \varepsilon \frac{\tan^2 \frac{\phi}{2} \hat{\kappa}_z \hat{g} \hat{\kappa}_z}{1 - \tan^2 \frac{\phi}{2} (\hat{\kappa}_z \hat{g})^2}$$

Yu-Shiba-Rusinov states:  $E_B = \Delta \frac{1-\gamma^2}{1+\gamma^2}$  with scattering strength  $\gamma = \pi S J N_0$

Mapping to circuit model yields:  $\gamma = \tan \frac{\delta\phi}{4}$  and  $E_B = \Delta \cos \frac{\delta\phi}{2}$

→ Scattering at FI has a similar effect as at strong magnetic impurities



Effective exchange field (a,b)

$$h_{eff} = \varepsilon \sin \frac{\phi}{2}$$

AG scattering rate

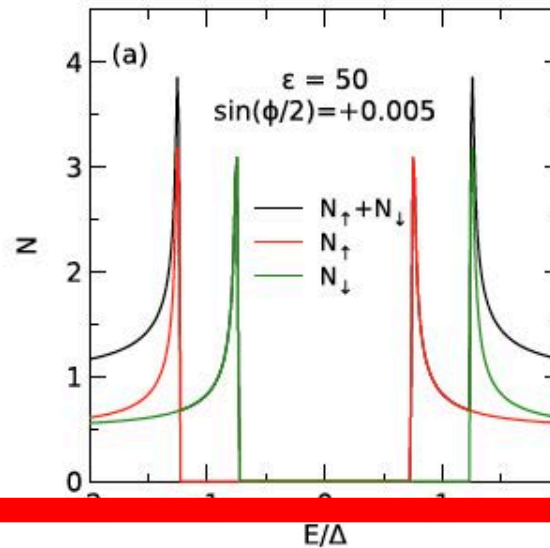
$$\Gamma_{AG} = \varepsilon \sin^2 \frac{\phi}{2}$$

Shiba energy

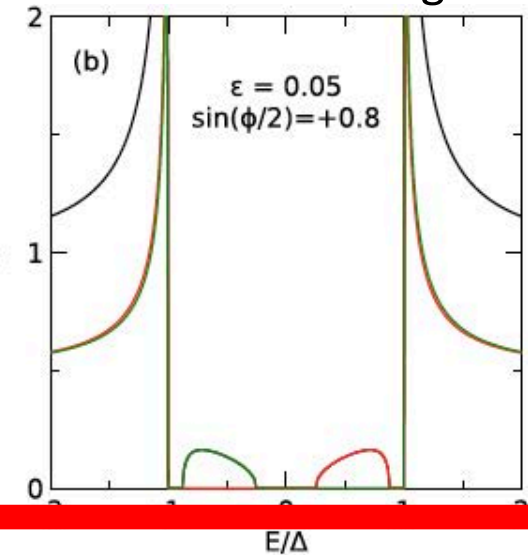
$$E_B = \Delta \cos \frac{\phi}{2}$$

[WB, Beckmann, JMMM in press]

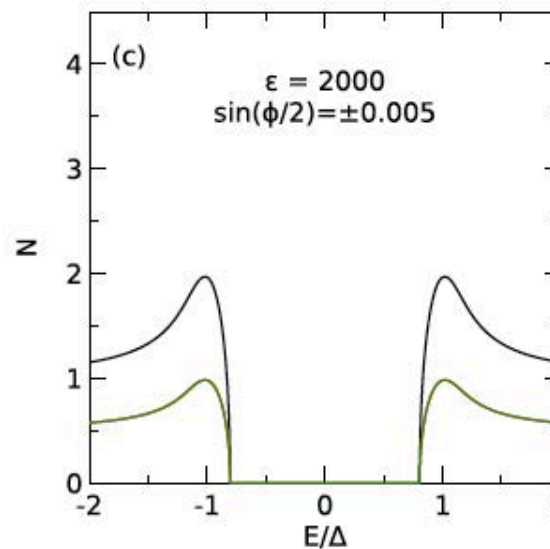
### Effective Field



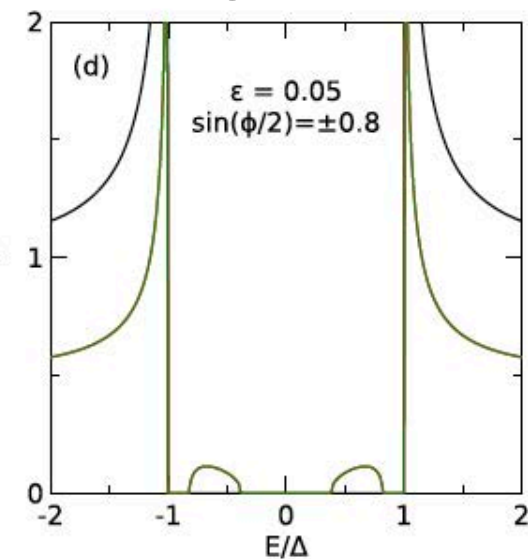
### Polarized Shiba regime



### Abrikosov-Gorkov regime



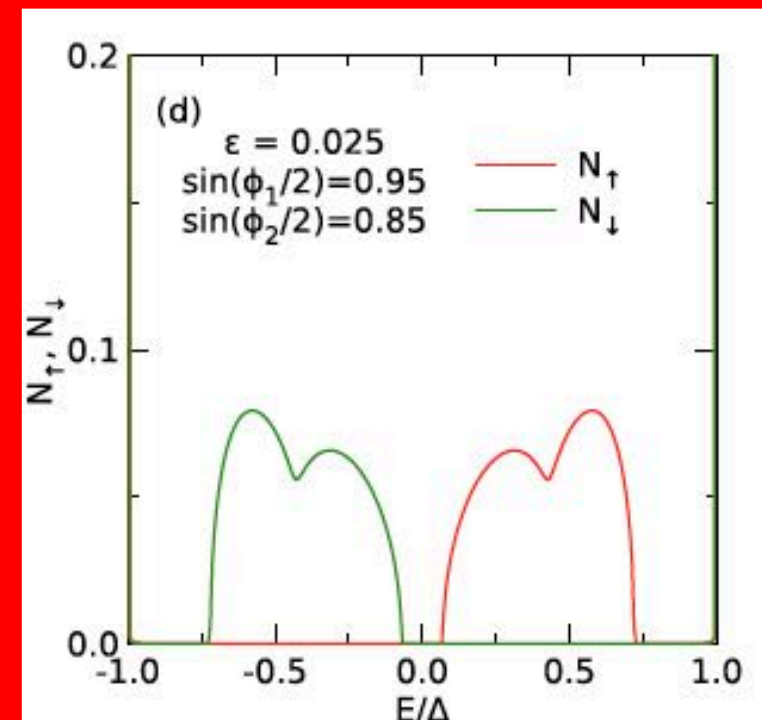
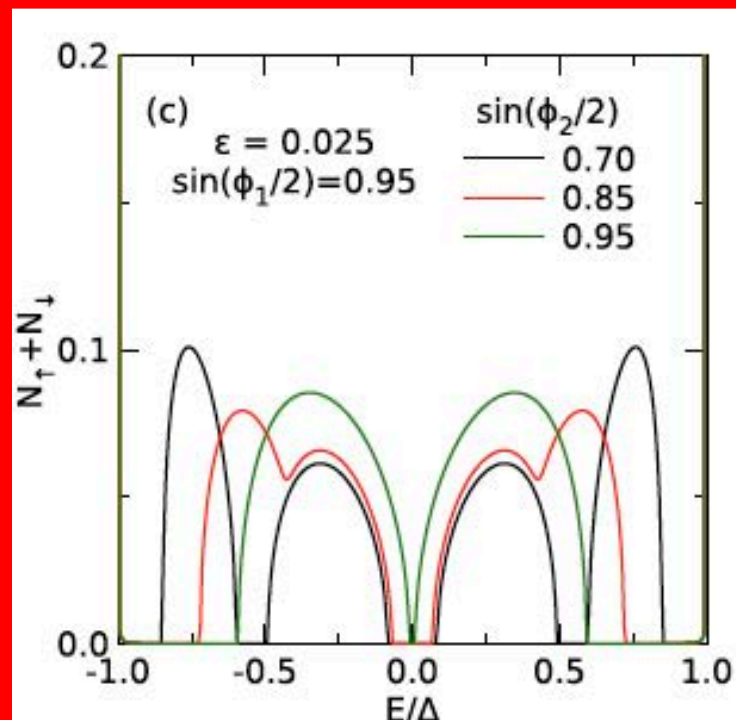
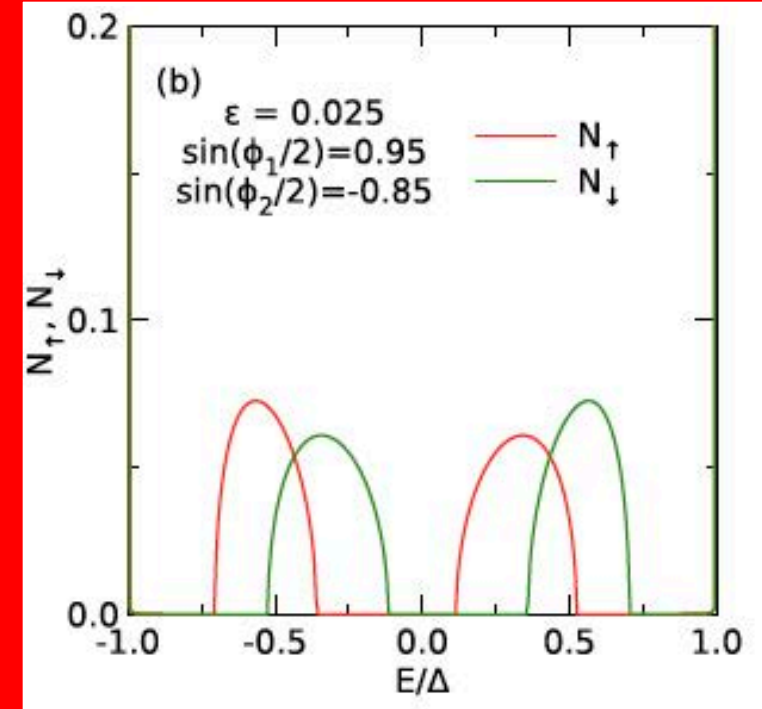
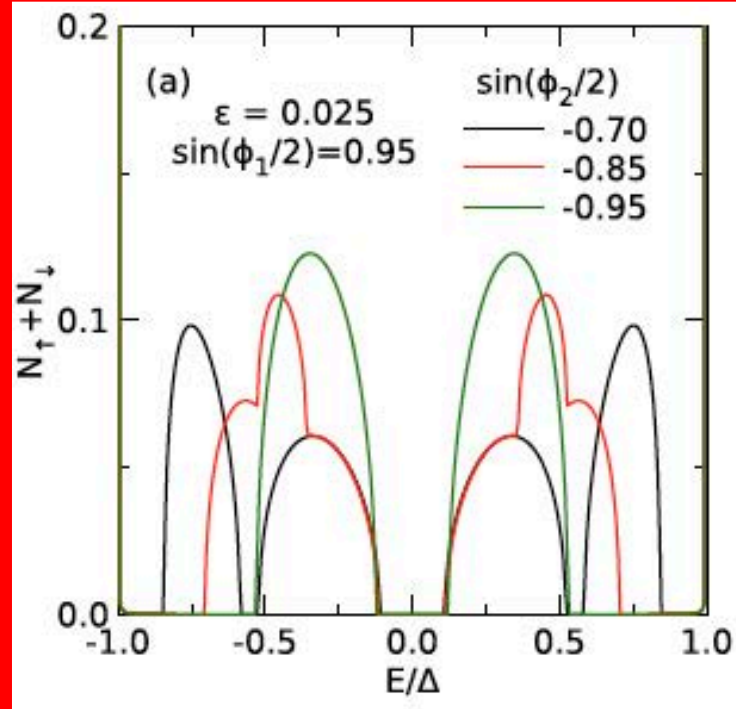
### Shiba regime



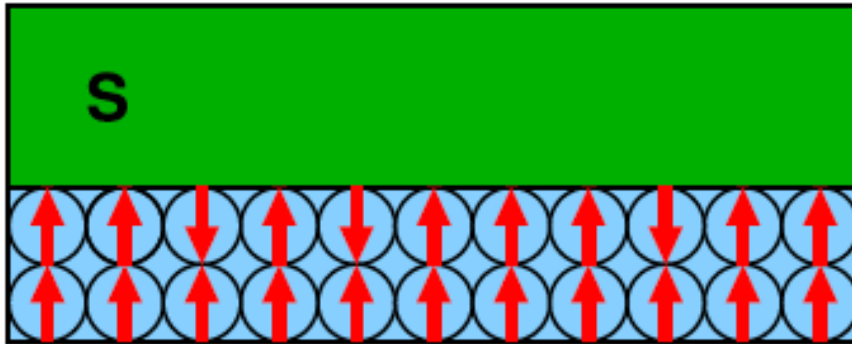
## Interacting Shiba bands

2 sublattices with  $\varepsilon_1 = \varepsilon_2 = \varepsilon$  with **different** spin mixing angles  $\phi_{1/2}$

- Orthogonal Shiba bands
- Revealed in spin polarised DOS
- Hybridized Shiba bands
- Spin polarized Shiba bands

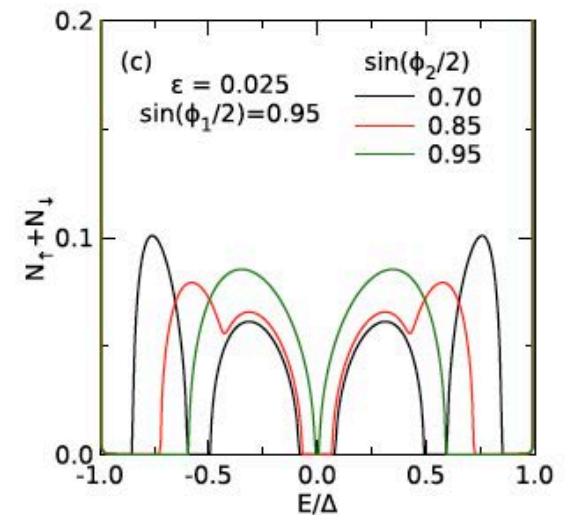
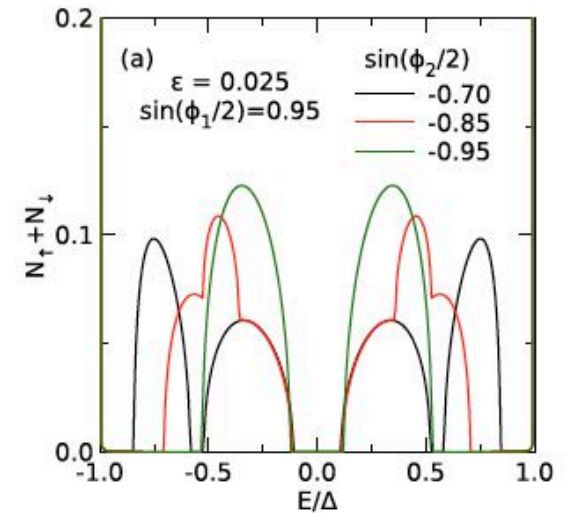






### Spin-dependent boundary effects in superconductors

- Formation of Shiba bands by strong spin scattering
- Spectroscopic signatures in the subgap density of states
- Opportunity for spin-polarized transport?



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